# The Heteroscedasticity Impact on Actuarial Science: Lee Carter Error Simulation

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## Abstract

**C19** Life insurance companies deal with two fundamental types of risks when issuing annuity contracts: financial risk and demographic risk. As regards the latter, recent work has focused on modelling the trend in mortality as a stochastic process. A popular method for modelling death rates is the Lee-Carter model. In this paper we gives an overview of the Lee Carter model and the feasibility of using it to construct mortality forecast for the population data. In particular, we focus on a sensitivity issue of this model and in order to deal with it, we illustrate the implementation of an experimental strategy to assess the robustness of the LC model. The next step, we experiment and apply it to a matrix of mortality rates. The results are applied to a pension annuity. There are investigating in particular the hypothesis about the error structure implicitly assumed in the model specification, after having assume that errors are homoscedastic. Analyzing the model it is estimated that the homoscedasticity assumption is quite unrealistic, because of the observed pattern of the mortality rates showing a different variability at different ages. Therefore, there is an emerging opportunity to analyze the strength of predictable parameter. The purpose of this study is a strategy in order to assess the strength of the Lee-Carter model inducing the errors to satisfy the homoscedasticity hypothesis. The impact of Lee Carter model on various financial calculations is the main focus of the paper. Furthermore, it is applied it to a matrix of mortality rates including a pension rate portfolio. The Albania model with the variables of death and birth is shown on this paper taken in consideration the Lee Carter Error.

**Keywords**: Lee-Carter model, mortality forecasting, Life Expectancy at Birth, Experimental Strategy, Singular Value Decomposition (SVD), Homoscedasticity.

## 1. Introduction

Life expectancy depends on several factors, which are age, gender, geographical region, and social class. Although some hypotheses for a long term change to the trend in life expectancy, due to causes such as obesity and worsening environmental conditions, it seems that life expectancies are following, and are likely to continue to follow, an increasing trend. It is assumed that less clear is the shape of this trend, particularly in light of two elements which emerge from empirical observations: life expectancies have increased at a faster rate than anticipated and the uncertainty in the projections of life expectancy increases as the time horizon increases. These ingredients are a source of concern to the financial institutions which provide financial products to the elderly such as pension annuities. If these trends are not considered, this type of contract will last longer than expected, bringing higher costs to these institutions. The situation can be summarized by saying that contracts like pension annuities are seriously exposed to longevity risk, and this exposure needs to be understood and managed. Further, it is well known that insurers generally use their internal models when major business decisions are to be made; for example pricing, product development and business planning all require the use of a pre-specified internal model.

After this, it can see that internal models play an important role in current actuarial practice. From a strict risk analysis viewpoint, the different risk aspects connected with the forecasting of future lifetimes arise from both accidental and systematic deviations of the actual data from their expected values. The longevity risk component is due to the systematic deviations of the deaths from the anticipated values, while the accidental variability in mortality causes a risk strongly influenced by the portfolio size. Working with a portfolio large enough to captivate the accidental demographic uncertainty, the demographic risk can be considered as constituting the systematic component and this is the main element that interacts with the risk arising from the variability in financial markets, the financial risk. The effect of both of these risks may be significant if we are referring to portfolios of life contracts of long duration such as pension annuities, which are characterized also by a multiplicity of payments. The consideration of the systematic aspect of the demographic risk is deepened in this paper by our focusing on the evolution over time of mortality rates. Continuing improvement in adult age mortality and in particular, during the most recent 30 years, this decreasing trend has been accelerating. Potential discontinuities in longevity trends have been taken into account by some demographers but most forecasters seem to think that discontinuities or jumps with respect to recent trends are to be considered extremely unlikely. The Lee-Carter model (Lee & Carter, 1992) has become a popular method for modelling and forecasting the trends over time in death rates: the main reasons are that it outperforms other models with respect to its prediction errors and that it is easy to implement. This methodology has become widely used in years and there have been various extensions and modifications proposed in order to attain a broader interpretation and to capture the main features of the dynamics of mortality rates, e.g. the log-bilinear Poisson version of the Lee-Carter model. In order to improve the survival probability outputs from the point of view of measuring uncertainty and hence longevity risk, recent approaches have used simulation procedures applied to the Lee-Carter family of models.

These studies provide prediction intervals for the forecasted quantities that are derived by using simulation techniques: this is an important feature because of the non-linear nature of the quantities under consideration. The approaches are based on the bootstrap methodology for obtaining more reliable and accurate mortality projections and utilize the semi parametric bootstrap based on the Poisson distribution. Furthermore, by nesting the original Poisson LC within the bootstrap with stratified sampling, we seek to reduce the approximation error for the statistics of interest. A sampling variance reduction approach for bootstrap mortality estimation is developed and analyzed in comparison with the bootstrap procedure as in Brouhns et al., 2005 (referred to herein as the Standard Procedure or SP). In this framework, the aim of the paper is to perform a new approach for describing the survival phenomenon by means of an experimental simulation approach applied to the LC model: the Stratified Sampling Bootstrap (SSP). The approach provides good results compared with the Standard Bootstrap and the Iterative Procedure (IP) proposed by Renshaw & Haberman (2003c) in respect of the Italian population. It is assumed that the demographic system for modelling a portfolio cash flow distribution impacts on key quantities such as technical reserves, surplus and funding ratios and it affects some of the choices in the management of pension plans. The analyze in this paper has been focused on the funding ratio, defined as the ratio between the market value of the assets and that of the liabilities at a certain time, and is chosen as a measure of the solvency of the insurance portfolio at that time. In particular, the purpose of this study, considering the results as measures of the cost of increased longevity, is to have a look on the financial implications of the improvement over time in mortality rates on the funding ratio. In this analysis, the financial risk is stochastically modelled and interacts with the demographic risk. Although recent econometric researchers highlight that a certain degree of correlation between these two sources of risk may exist, the risks are assumed to be independent in both the classic actuarial context of pricing and reserving in life insurance. In this paper, valuations are performed at the time of issue on the basis of the financial and demographic information and are highly sensitive to the strength of the longevity phenomenon. Survival modelling is required, from the pension plan point of view, for the estimation of the premiums expected to be received during the accumulation period up to retirement age, and of the benefits expected to be paid during the period from retirement age until the death of the policyholder. In the numerical examples, it will discussed funding ratio values using the SSP and compare them with those that result from the other two techniques under consideration: SP and IP, in a context in which interest rates are stochastically modelled. After that, the analysis is completed with the calculation of the risk arising from the uncertainty in the choice of the simulation approach: this is allowed for by assigning probabilities to the choice of each of the three different approaches, in light of the subjective degree of reliability that the insurer attaches to each method.

The contextual of the research is based on the bilinear mortality predicting methods. These methods are taken into account to describe the enlargements in the mortality trend and to project survival tables. The focus is based on Lee-Carter method for modelling and forecasting mortality. In particular, a sensitivity subject of this model and in order to deal with it, it is illustrated the implementation of an experimental strategy to assess the robustness of the LC model. The results are applied to a matrix of mortality rates in a pension annuity portfolio.

The LC method is an influential approach to mortality forecasting rate in different population. In analyzing these forecasting models, it is allowed to ensure that the most appropriate random process is used for each mortality model. In this study it is consider a variety of processes including multivariate, autoregressive process etc. The data included in this paper are from Italian pension annuity portfolio and Albania data mortality and birth. Below you may find the traditional LC model analytical expression:

$$\ln(Mx,t) \quad = ax + bx\ Kt + Ex,t \ (1)$$

This equation describe the logarithm of a time series of age-specific death rates mx,t as the sum of an age-specific parameter that is independent of time αx and a component given by the product of a time-varying parameter κt. In this expression it is clearly reflecting the general level of mortality and the parameter βx who is representing the trend of mortality at each age of group, how highly or slowly is generated when the general level of mortality changes.

The term Ex,t is the error term, assumed to be homoscedastic with mean 0 and variance σ2. Now it is calculated the same equation (1), if Mx,t is the matrix holding the mean centered log-mortality rates. It is seen that the LC model can be expressed as per below:

$$\mathrm{Mx,t} = \ln(Mx,t) \quad - ax = bx\ Kt + Ex,t \ \ (2)$$

Following Lee Carter model (LC), the parameters βx and κt can be estimated according to the Singular Value Decomposition (SVD) with suitable normality restrictions. The LC model incorporates different sources of uncertainty in the demographic model and uncertainty in forecasting. The former can be incorporated by considering errors in fitting the original matrix of mortality rates, while forecast uncertainty arises from the errors in the forecast of the mortality index. In our contribution, we deal with the demographic component in order to consider the sensitivity of the estimated mortality index. In particular, the research consists in defining an experimental strategy to force the fulfilment of the homoscedasticity hypothesis and evaluate its impact on the estimated κt.

## 2. Lee Carter Model specification

The experimental strategy introduced above, with the aim of inducing the errors to satisfy the homoscedasticity hypothesis, consists in the following phases. The error term can be expressed as follows:

$Ex,t = Mx,t - bx\ Kt$ (3)

As the difference between the matrix Mx,t, referring to the mean centered log mortality rates and the product between βx and κt deriving from the estimation of the Lee Carte model (LC). The successive step consists in exploring the residuals by means of statistical indicators such as: range, interquartile range, mean absolute deviation (MAD) of a sample of data, standard deviation, box-plot, etc. After that, it proceed with finding those age groups that show higher variability in the errors. Once it is explored the residuals Ex,t, it may find some non-conforming age groups. For each selected age group, it is possible to reduce the variability by dividing the entire range into several quantiles, leaving aside each time the fixed α% of the extreme values. It is replicate each running under the same conditions a large number of times (i.e., 1000). For each age group and for each percentile, it is defined an error matrix. The fitted parameter figures strongly depend on the constraints enforced, which becomes more obvious in the graphs that follow. This implies that the constraints are a determining factor when deciding what time series process to use for generating mortality scenarios. In a multi-population model, we expect a certain behavior from its parameters. In a perfect situation, the common parameters should be able to capture the true global mortality trend, in both age and time, amongst the populations as a whole. If the underlying philosophy of the model is correct, then we would expect that the country-specific period effects all data around some constant level in the long term. Significant differences from this level, in either range or shape would mean that this particular population is somehow different from the other populations and there should be a reason for this behavior. The maximum-likelihood estimation is highly influenced by the population size and the countries with bigger size should have a higher impact when estimating the common mortality parameters. The successive running give more and more homogeneous error terms.

First we have the Lee Carter model, where it is estimated the parameters αx, βx and Kx. The parametrization of variables is like ∑kj=0 and $\sum_{k=0}^{n} k^2 = 1$. Under this model, mortality projections are obtained by projecting a time series for kj. By way of this experiment, it is investigated the residual's heteroscedasticity deriving from two factors: the age group effect and the number of altered values in each age group. In particular, the focus is to determine the hypothetical pattern of κt by increasing the homogeneity in the residuals. Thus, under these assumptions, it is analyzed the changes in κt that can be derived from every simulated error matrix. In particular, at each running it is obtained a different error matrix Ex,t, which is used for computing a new data matrix Mx,t, from which it is possible to derive the correspondent κt. To clarify the procedure analytically, we will see the following relation:

$$Mx,t - Ex,t = Mx,t \overrightarrow{\quad} bx\ Kt\ (4)$$

where Mx,t is a new matrix of data obtained by the difference between Mx,t (the matrix holding the raw mean centered log mortality rates) and Ex,t (the matrix holding the mean of altered errors).From Mx,t, if βx is fixed, it is obtain the κt as the ordinary least square (OLS) coefficients of a regression model. The procedure is replicated by considering further non-homogenous age groups with the result of obtaining at each step a new κt. It is running a graphical exploration of the different κt patterns explained below. Thus, it is plot the experimental results so that all the κt's are compared with the ordinary one. Moreover, the slope effect of the experimental κt is compared through a numerical analysis.

The experiment is focused on the Italian data matrix in log mortality rates as an experiment, for the male population from 1950 to 2000. In particular, the rows of the matrix represent the 21 age groups [0], [1–4], [5–9], ..., [95–99] and the columns refer to the years 1950–2000. The procedure consists of an analysis of the residuals' variability through some dispersion indices which help to determine the age groups in which the model hypothesis does not hold. It is noticed that the residuals in the age groups 1–4, 5–9, 15–19 and 25–29 are far from being homogeneous. Thus the age groups 1–4, 15–19, 5–9, 25–29 will be sequentially, and according to this order, entered in the experiment. Alongside the dispersion indices, it is provided a graphical analysis by displaying the box plot for each age group, where on the x-axis the age groups are reported and on the y-axis the residuals' variability. If we look at the age groups 1–4 and 15–19 can notice that they show the widest spread compared to the others. In particular, we perceive that for those age groups the range goes from−2 to 2. For this reason, there was explore to what extent the estimated κt are affected by such a variability. A way of approaching this issue can be found by means of the following replicating procedure, implemented in a Mat lab routine. For each of the four age groups it is substitute the extreme residual values with the following six quantiles: 5%, 10%, 15%, 20%, 25%, 30%. Then it is generated 1000 random replications (for each age group and each interval). From the replicated errors (1000 times × 4 age groups×6 percentiles) it is compute the estimated κt (6×4×1000times) and then the 24 averages of the 1000 simulated κt. The 24,000 estimated κt through a Plot-Matrix, representing the successive age groups entered in.

| Age | IQ Range | MAD | Range | STD |
|---|---|---|---|---|
| 0 | 0.107 | 0.059 | 0.300 | 0.075 |
| *1–4* | *2.046* | *0.990* | *4.039* | *1.139* |
| *5–9* | *1.200* | *0.565* | *2.318* | *0.653* |
| 10–14 | 0.165 | 0.083 | 0.377 | 0.099 |
| *15–19* | *1.913* | *0.872* | *3.615* | *1.007* |
| 20–24 | 0.252 | 0.131 | 0.510 | 0.153 |
| *25–29* | *0.856* | *0.433* | *1.587* | *0.498* |
| 30–34 | 0.536 | 0.250 | 1.151 | 0.299 |
| 35–39 | 0.240 | 0.186 | 0.868 | 0.239 |
| 40–44 | 0.787 | 0.373 | 1.522 | 0.424 |
| 45–49 | 0.254 | 0.126 | 0.436 | 0.145 |
| 50–54 | 0.597 | 0.311 | 1.290 | 0.367 |
| 55–59 | 0.196 | 0.151 | 0.652 | 0.187 |
| 60–64 | 0.247 | 0.170 | 0.803 | 0.212 |
| 65–69 | 0.207 | 0.119 | 0.604 | 0.147 |
| 70–74 | 0.294 | 0.171 | 0.739 | 0.202 |
| 75–79 | 0.230 | 0.117 | 0.485 | 0.133 |
| 80–84 | 0.346 | 0.187 | 0.835 | 0.227 |
| 85–89 | 0.178 | 0.099 | 0.482 | 0.124 |
| 90–94 | 0.307 | 0.153 | 0.701 | 0.186 |
| 95–99 | 0.071 | 0.042 | 0.220 | 0.051 |

*Table 1 Different dispersion indices to analyze the residuals variability (Source: Authors D'Amato, Russolillo)*

After this analyze we can see the crude mortality rates and the model fits under the various models. The crude mortality rates are shown as open circles, while the original Lee-Carter parameters are shown as a dark grey line. The DDE parameters are shown by a light grey line and the CR parameters are shown by a dashed line. The original Lee-Carter parameters are largely obscured by the DDE line, due to them being almost completely co-incident, i.e. the LC and DDE fits are almost identical. As we see, at the age 40 the log mortality is more dispersed considering the log mortality at 60, 70 and 80. At the years 1960-1980 the Lee carter rates and DDE rates are distant from each other, while in recent years this curbs tends to be closer.

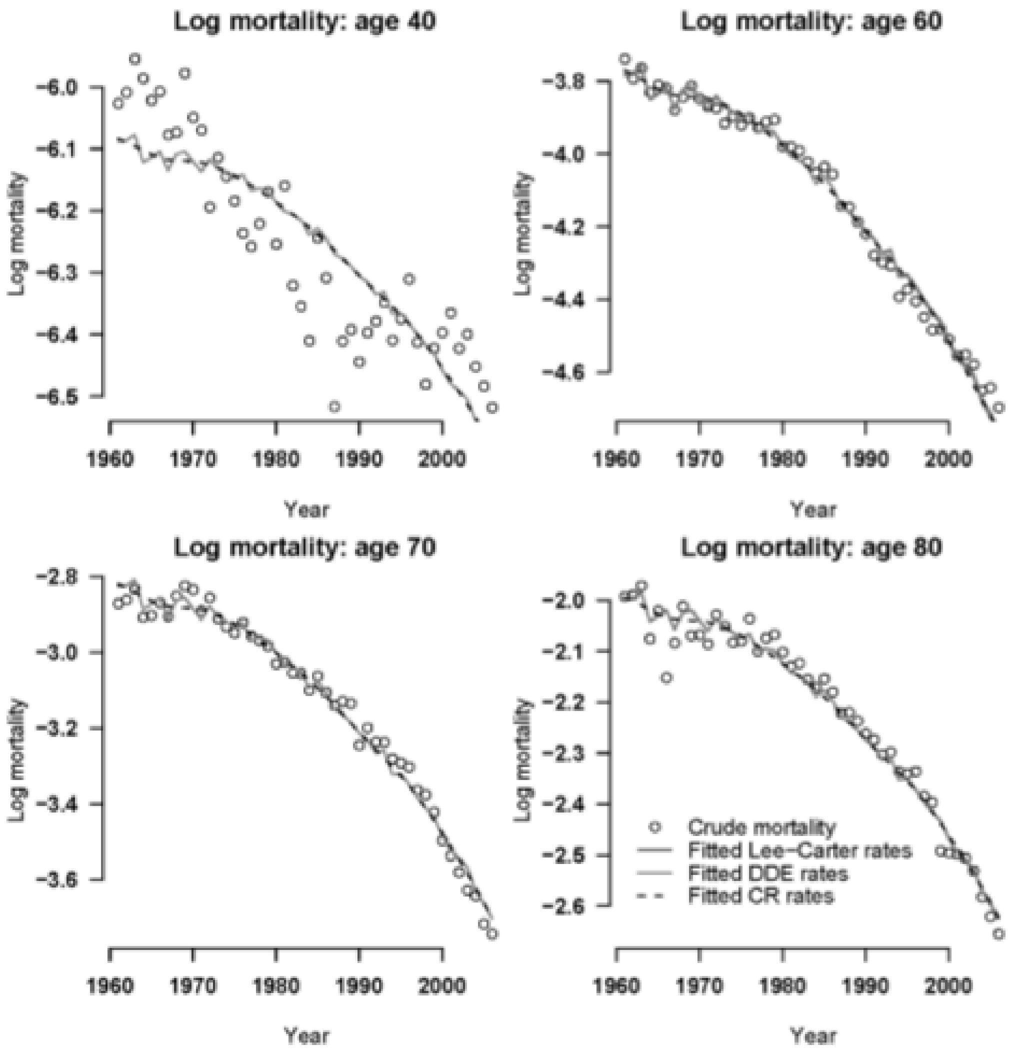


*Fig.1 Log mortality at selected ages the dark grey line for original Lee-Carter parameters is largely obscured by the DDE line due to them being almost completely co-incident, i.e. the LC and DDE fits are almost identical (Source: Authors Richards, Currie)*

Parameter plots for ax, bx and ky (left column), and the same parameters after linear adjustment (right column); the original Lee-Carter parameters are shown as solid dots, while the DDE parameters are shown by a solid line and the CR parameters are shown by a dashed line; the linear-adjusted plots show the same coefficients on the left after subtracting a fitted straight line; they show, for example, that the pattern of ax by age is not as linear as it seems (ONS data) the original Lee-Carter model. In each case there is an obvious smooth pattern in the parameters, hence the extension of the DDE and the CR models to smooth bx (DDE and CR) and ky (CR only). The crude mortality rates are shown as open circles, while the original Lee-Carter parameters are shown as a dark grey line. The DDE parameters are shown by a light grey line and the CR parameters are shown by a dashed line. The original Lee-Carter parameters are largely obscured by the DDE line, due to them being almost completely co-incident, i.e. the LC and DDE fits are almost identical. The projected log-mortality at age 65 under the DDE and CR approaches. While the central projections are very different, we can see that the confidence bounds substantially overlap, suggesting that the projection from one model is quite consistent with the projection from the other. It is interesting that the original Lee-Carter model has a confidence area which is essentially the more pessimistic half of the confidence area for the CR model. One notable feature is that the CR model has a much wider confidence interval.

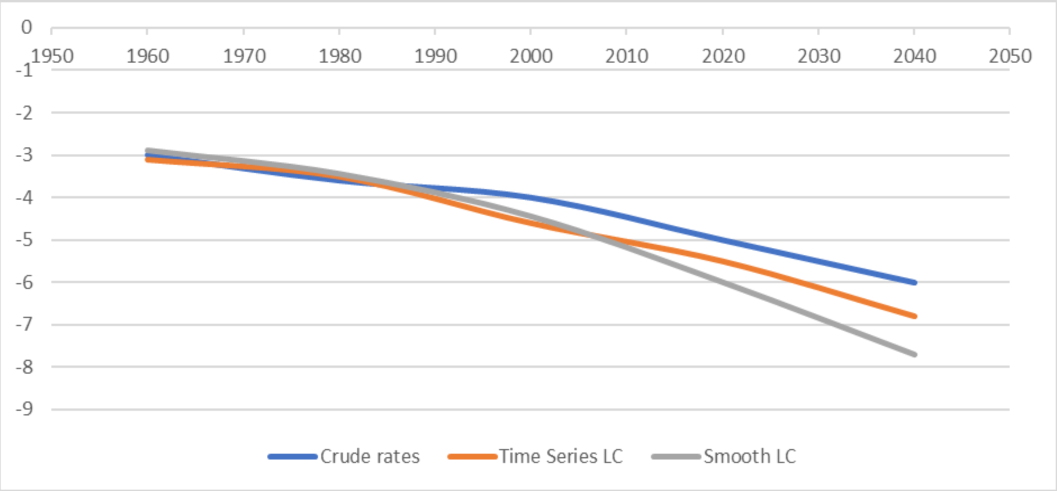


*Fig.2 Mortality forecast at age 65 with 95% confidence intervals; the solid grey line is the time-series Lee-Carter forecast, together with shaded 95% confidence area; the dashed line is the smoothed-Lee-Carter forecast, together with 95% confidence bounds. (Source: Authors D'Amato, Russolillo)*

In the figure we see that from1960 to 2010, Crude rates, Time series LC and Smooth LC tend to be closed, without dispersion from each other, while after 2010 this curbs tends to go in the separate mode.

In contrast, with smaller data sets it is hard to prove the existence of more complicated patterns, thus leading to only simplistic models being fitted with narrow confidence bands, which might give an illusion of certainty. This paradox is discussed in greater detail in CMI (2005).

In below we can analyze the graph with Albanian data population. As we see the difference data birth from death is extremely lower in the recent years.

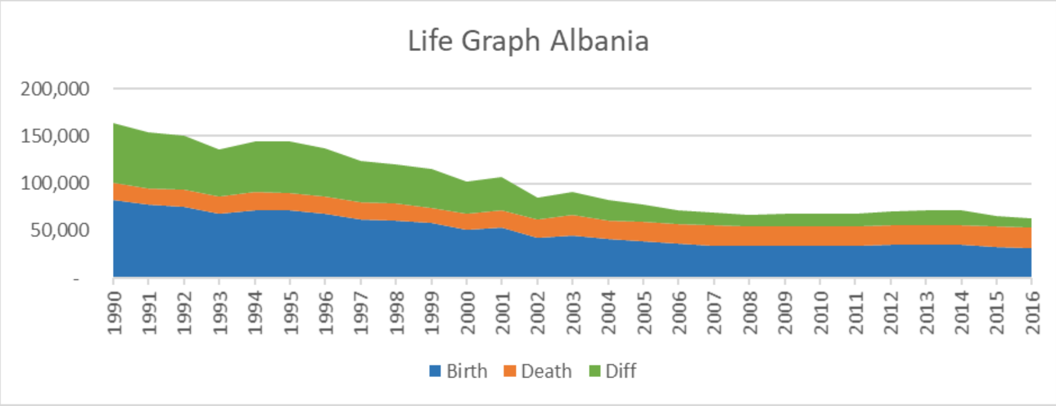


*Fig.3 The graph of Albania population- birth and death per years (Source: INSTAT, Albania, O. Idrizi analyze)*

From the graph we can clearly identify that the number of death, taking in consideration the number of birth is increased with a difference from 63,932 to 10,345 (birth – death). We have also analyzed that the number of divorce considering the marriage is lower with years.

This study include information by different institutions like Ministry of Finance, public and private pension institutes, INSTAT (Statistic institution) and other institutions serving in this field. It is analyzed the economic development of the country, as well human resources, social, economic and natural, in the economic development; to assess the sources of contributions and the contributors rights; to study the performance of the social security system along transition period with a view to identifying obstacles and ways to overcome them; to provide a clear picture of the demand for development of the pension market. Pension system in Albania is facing a wide variety of interrelated problems. Albania has experienced a high level of informal labor markets as the contributions are low and this situation has brought to limited income from the wage contributions within the country itself. A large percentage of the working population is not making contributions and there are fewer workers that support the older age even nowadays. For instance, the 'urban' people have to pay high contributions for limited pensions and non-urban people pay low contribution to take low pensions, which is overall relatively high in comparison to what they should pay. Furthermore, low rates of participation in the system suggests that when these workers retire a lower percentage of the elderly which will be covered by the pension system. Significantly, when workers with few contributions reach the retirement age, they will be ineligible for benefits, which show an increase of the number of elderly facing poverty in the nearer future. According to the data presented by the World Bank, between 35 to 50 percent of the future elderly will have no pensions, compared to nowadays where almost all elderly persons can set-up and collect pensions. The estimated parameters at the Table 2 are shown in Figure3.

| | Birth | Death | Diff |
|---|---|---|---|
| 1990 | 82,125 | 18,193 | 63,932 |
| 1991 | 77,361 | 17,743 | 59,618 |
| 1992 | 75,425 | 18,026 | 57,399 |
| 1993 | 67,730 | 17,920 | 49,810 |
| 1994 | 72,179 | 18,342 | 53,837 |
| 1995 | 72,081 | 18,060 | 54,021 |
| 1996 | 68,358 | 17,600 | 50,758 |
| 1997 | 61,739 | 18,237 | 43,502 |
| 1998 | 60,139 | 18,250 | 41,889 |
| 1999 | 57,948 | 16,720 | 41,228 |
| 2000 | 51,242 | 16,421 | 34,821 |
| 2001 | 53,205 | 19,013 | 34,192 |
| 2002 | 42,527 | 19,187 | 23,340 |
| 2003 | 45,313 | 21,294 | 24,019 |
| 2004 | 40,989 | 20,269 | 20,720 |
| 2005 | 38,898 | 20,430 | 18,468 |
| 2006 | 35,891 | 20,852 | 15,039 |
| 2007 | 34,448 | 20,886 | 13,562 |
| 2008 | 33,445 | 20,749 | 12,696 |
| 2009 | 34,114 | 20,428 | 13,686 |
| 2010 | 34,061 | 20,107 | 13,954 |
| 2011 | 34,285 | 20,012 | 14,273 |
| 2012 | 35,473 | 20,870 | 14,603 |
| 2013 | 35,750 | 20,442 | 15,308 |
| 2014 | 35,760 | 20,656 | 15,104 |
| 2015 | 32,715 | 22,418 | 10,297 |
| 2016 | 31,733 | 21,388 | 10,345 |

*Table 2 Albania population data on birth and death per years (Source: INSTAT, Albania, O. Idrizi analyze)*

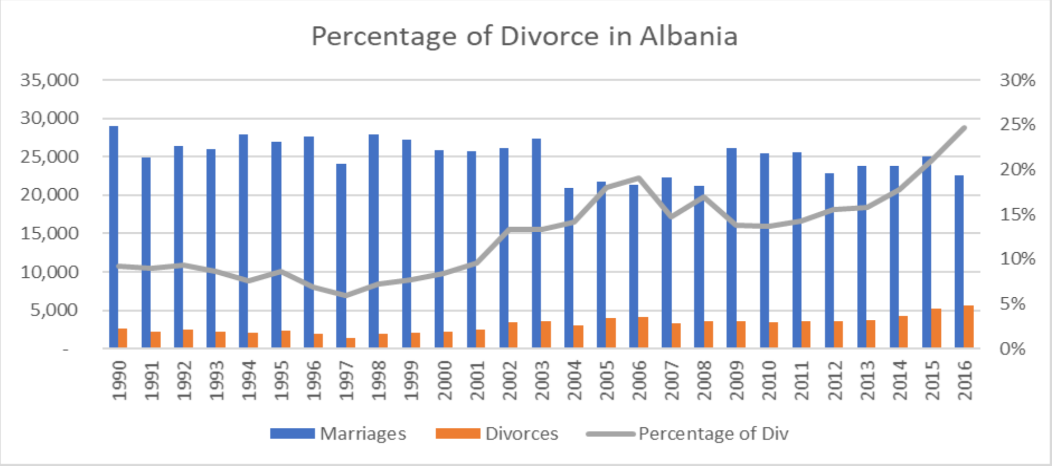

*Fig.4 Graph of marriage and divorces in Albania population (Source: INSTAT, Albania, O. Idrizi analyze)*

The age group and the different percentiles effect the experiment in the four rows and the successive increment in the percentage of outer values.

It is noticed that the different κt behavior in the four rows as more age groups and percentiles are considered. For better interpretation of these results, it is strategized a synthetic view of the resulting average of the 1000 κt under the 24 conditions and compared them with the series derived by the traditional LC estimation.

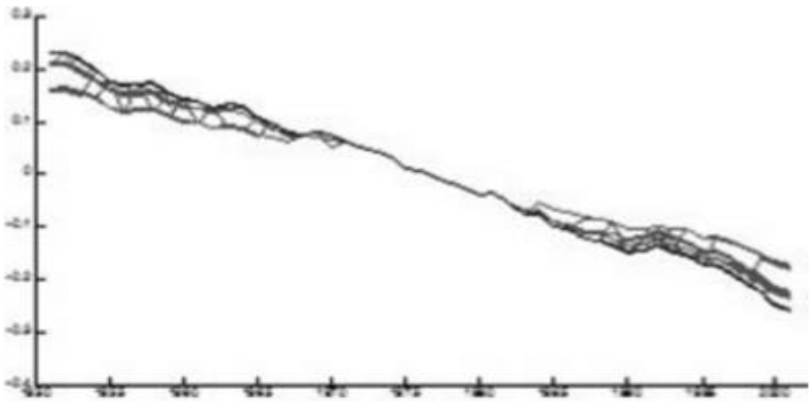

Fig. 5 .A comparison between the 24 averaged κt (in red) and the original one (in black) *(Source: Authors D'Amato, Russolillo)*

In Figure 5, where on the x-axis there are the years from 1950 to 2000 and on the y-axis there are the κt values, it is represent the 24,000 κt grouped according to the 24 different experimental conditions. The impact on the κt series of the age groups change and of the increase of percentage of random values considered in the selected age groups. It is noticed that the κt derived by the experiment (in red) tends to be flatter than the original one (drawn in black), i.e., there are changes in homogeneity on the κt for each of the four age groups. By comparing the ordinary κt to the simulated ones, it is obtained information about the effect of the lack of homoscedasticity on the LC estimates. To what extent does it influence the sensitivity of the results? It is assumed that the more homogenous the residuals are, the flatter the κt is. From an actuarial point of view, the κt series reveals an important result: when it is used the new κt series to generate life tables, it is see survival probabilities lower than the original ones. The effect of that on a pension annuity portfolio will be illustrated in the following application.

In this section, it is provided an application of the previous procedure for generating survival probabilities by applying them to a pension annuity portfolio in which beneficiaries enter the retirement state at the same time. In particular, having assessed the breaking of the homoscedasticity hypothesis in the Lee-Carter model, we intend to quantify its impact on given quantities of interest of the portfolio under consideration. The analysis concerns the dynamic behavior of the financial fund checked year by year arising from the two flows in and out of the portfolio, the first consisting in the increasing effect due to the interest maturing on the accumulated fund and the second in the outflow represented by the benefit payments due in case the pensioners are still alive. Let us fix one of the future valuation dates, say corresponding to time κ, and consider what the portfolio fund is at this valuation date. As concerns the portfolio fund consistency at time κ, we can write:

$$Zk = Zk - 1\ (1 + ik^{*}) + N^{K}P \quad with \quad k = 1, 2\ , \ldots.., n - 1\ (5)$$

$$Zk = Zk - 1\ (1 + ik^{*}) + N^{K}R \quad with \quad k = n, n + 1\ , \ldots.., w - x\ (6)$$

Where N0 represents the number of persons of the same age x at contract issue t =0 reaching the retirement state at the same time n, that is at the age x +n, and i∗ κ is a random financial interest rate in the time period (k −1, k). The formulas respectively refer to the accumulation phase and the annuitisation phase.

Problems Identify: First, the process of parameter estimation might experience convergence problems if identifiability is not addressed. Second, even if θ and forecast θ are parameterization that give identical historical fits, forecast distributions of mortality rates might be different. The second point means that we need to take care when fitting, e.g. a time series model to the period effects to ensure that models allow in a consistent way for the identifiability problem.

## 3. Forecasting Mortality

Referring to the financial scenario, it refers to the interest rate as the rate of return on investments linked to the assets in which insurer invests. In order to compare, there are both considered a deterministic interest rate and a stochastic interest rate framework.

As regards the former, it is assumed that the deposited portfolio funds earn at the financial interest rate fixed at a level of 3%. As regards the latter, it is adopted the Vasicek model. This stochastic interest rate environment seems to be particularly suitable for describing the instantaneous global rate of return on the assets linked to the portfolio under consideration, because of potential negative values. As is well known, this circumstance is not in contrast with the idea of taking into account a short rate reflecting the global investment strategy related to the portfolio.

As concerns the mortality model, in this study it is considered the survival probabilities generated by the above-described simulation procedure and by the classical estimation of the Lee-Carter model (traditional method). In the former methodology considering is the κt series arising from the experiment. Following the Box-Jenkins procedure, ARIMA (0, 1, 0) model is more feasible for our time series. After obtaining the κt projected series, it is constructed the project life table and then generalize the probabilities referred to insured aged x = 45. In Figure 6 it is report the survival probability distribution as a function of different LC estimation methods: the traditional and the simulation methods by D'Amato and Russolillo. As we can see, the pattern of simulated probabilities lies under the traditional probabilities. Moreover this difference increases as the projection time increases. Thus, referring to the financial and the demographic stochastic environments described above, we evaluate the periodic portfolio funds. As regards the premium calculation hypotheses, there are used two different assumptions (simulated LC, classical LC) and the fixed interest rate at 4%. It is used the same mortality assumptions made in the premium calculation even for the portfolio fund dynamics from the retirement age on, i.e., which means to resort to a sort of homogeneity quality in the demographic

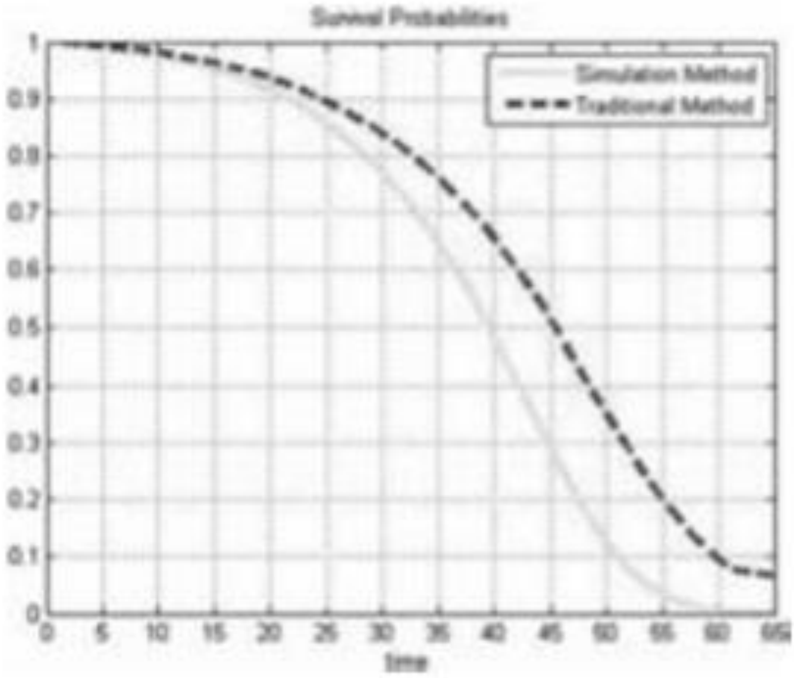


Fig.6 Comparison between the two different methods for generating survival probabilities on the basis of the Lee-Carter model: traditional and simulation method *(Source: Authors D'Amato, Russolillo)*

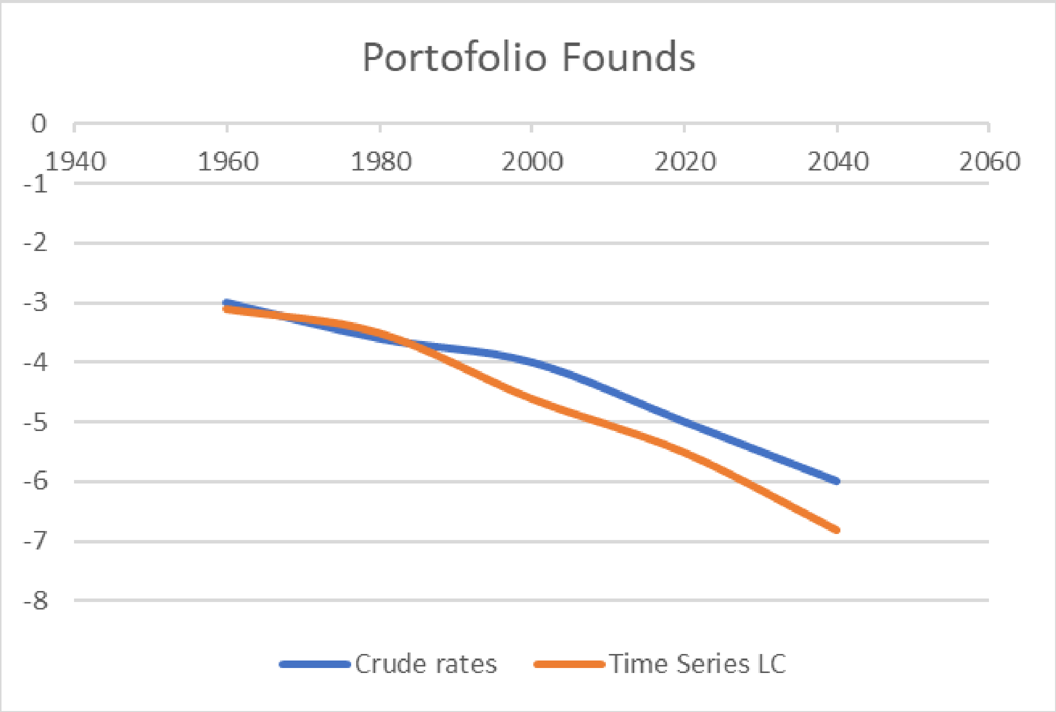


Fig. 7 Portfolio of 1000 pension annuities, x =45, t =20, r =100. Fixed rate at 3% *(Source: Authors D'Amato, Russolillo)*

In the following graphs (see Fig.6 and 7) were present the portfolio funds along with the potential whole contract life, i.e., both into the accumulation phase and into the annuitisation phase. The portfolio funds trend is calculated on a pension annuity portfolio referred to a cohort of c = 1000 beneficiaries aged x =45 at time t =0 and entering in the retirement state 20 years later, that is at age 65. The cash flows are represented by the constant premiums P, pay able at the beginning of each year up to t =20 in case the beneficiary is still alive at that moment (accumulation phase) and by the constant benefits R =100 pay able at the beginning of each year after t =20 (annuitisation phase) in case the beneficiary is still alive at that moment. Figure 7 shows how the portfolio funds increase with

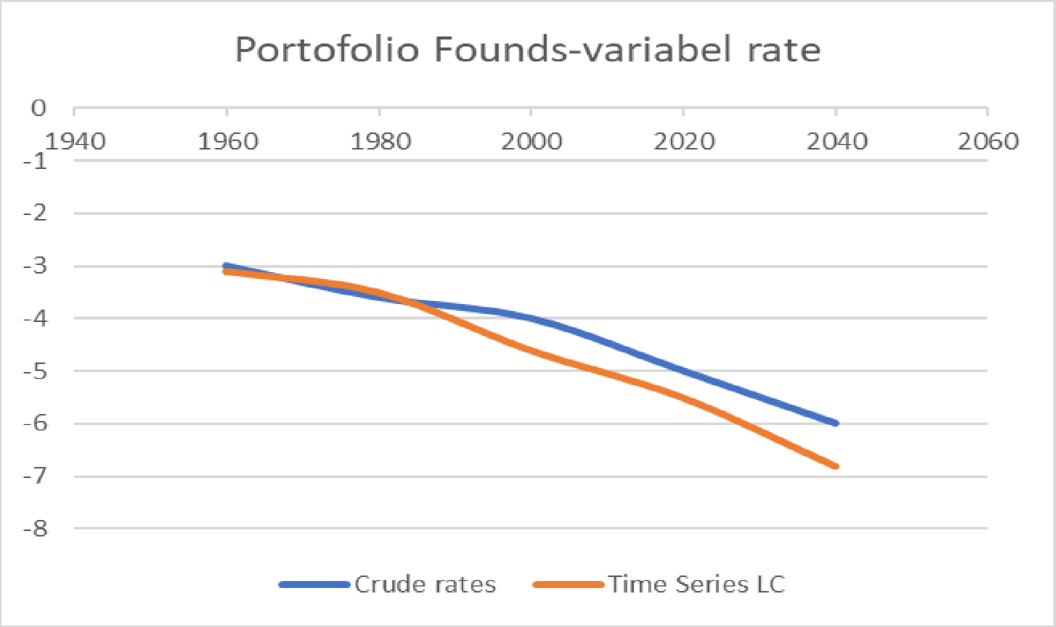


Fig.8 Portfolio of 1000 pension annuities, x =45,t =20,r =100. *(Source: Authors D'Amato, Russolillo)*

Stochastic rate of return better survival probabilities. In particular, in this figure is represented the portfolio funds earning interest, term by term, at the fixed rate of return of 3%, from the time issue on. As a first result it is find out that the portfolio fund amount is overestimated when the survival probabilities are calculated on the basis of the projection of the traditional LC estimation. On the basis of the results reported above, we can notice how the lack of homoscedasticity affects the portfolio risk assessment. Finally, it is evaluated the portfolio fund consistency from the contract issue on, adopting the Vasicek model for describing the instantaneous global rate of return on the assets linked to the portfolio under consideration. As in the previous case, Figure 8 shows that the traditional forecasting method blows up the portfolio funds amount both into the accumulation and into the annuitisation phases. The findings seen in this paper are confirmed also in the case of the stochastic rate of return. For this reason, it is provided the evidence that the lack of homoscedasticity has a strong effect on the actuarial results.

## 4. Conclusions

The simulation procedure proposed in this paper is characterized by an experimental strategy to stress the fulfilment of the homoscedasticity hypothesis of the LC model. In particular, it is simulated a different experimental conditions to force the errors to satisfy the model hypothesis in a fitting manner. Besides, there are developed the $\kappa t$ series for generating more realistic survival probabilities. In this study it is measured the impact of the two different procedures for generating survival probabilities, using the traditional and simulation methods, on a portfolio of pension annuity. The applications, referred to the male population, show that the probabilities generated on the basis of the simulation procedure are lower than the probabilities obtained through the traditional methodology by the LC model. In particular, if we apply the simulated projections to a financial valuation of periodic portfolio funds of pension annuity portfolio, we can observe lower corresponding values than the traditional one, in both the so called accumulation and annuitisation phases. In this paper it is compared the multi-population mortality model and two of its variants with the Common Age Effect (CAE) model by Kleinow (2015) using mortality data from two countries (Italia and Albania). Especially, we can notice more size able portfolio funds in the event of traditional methodology. In other words, the insurer's financial position would be overestimated by means of the traditional method in comparison with the simulation method. The results of the appraisal arise from the different behaviors of the residuals. In fact, in the traditional methodology, there is heteroscedasticity in the residuals for some age groups which can lead to more optimistic survival projections. On the other hand, on the basis of the simulation procedure, the final result shows how a more regular residual matrix leads to a flatter κt series according to the LC model hypothesis. This circumstance determines more pessimistic survival projections.

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