Low-Energy Quark Mass Test for *U* and *D* via Elastic Form Factors

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Abstract

The study was aimed at providing a device to estimate the range of values of the *u*- and *d*-quark masses through the elastic *ep*-scattering form factors at the low energy regime. ROOT generated *dcsep* data sets, from theoretical and experimental form factors, were compared to modified *dcseq* and their intersections were determined from the average of a total of 3000 events for each *dcs* at various scattering angles selected randomly from 0^{o} to 180^{o}. The proton mass was required as a parameter used in the relativistic recoil factor of *dcseq* to shift its distribution closer to *dcsep* thereby attaining the critical intersections. For quarks carrying effective masses, the extrapolated energy intersection of *dcsep* generated from the average of all form factors with the modified *dcseu* is $226.00013 MeV^2$ and this is lesser than that of the modified *dcsed* at $1093.00004 MeV^2$ with bin size of $1 MeV^2$ and their respective *dcs* intersections are 10.07049x10-4 and 0.36976x10-4, in barns. Summary of results are also given for quark masses derived from MS scheme and Lattice QCD. By considering all possible scattering angles at fixed transfer momentum, the relativistic recoil factor was treated as a constant that shifted the distribution and gave rise to a tool in estimating quark mass range.

Keywords: Quark Masses, Form Factors, *dcs*, Proton Mass, Relativistic Recoil Factor

Introduction

The up (*u*) and down (*d*) quarks are the lightest generation of quarks. As major constituents of matter, they form the proton (*uud*) and neutron (*udd*). Their respective masses, m u=2.2$^{+0,5}$- 0.4MeV and md=4.7$^{+0,5}$-0.3MeV, are estimates in a Mass-independent Subtraction (MS) scheme [19]. They were first observed by experiments at the Stanford Linear Accelerator Center (SLAC) in 1968 [3,5] through deep inelastic scattering (DIS) experiments which indicated the protons to be made-up of three of these fundamental substructures [8]. Despite being common, the bare masses of u and d are not well determined. However, Lattice QCD calculations have a more precise value of 2.01±0.14MeV/c^2 and 4.79±0.16MeV/c^2, respectively [7].

Masses of quarks are fundamental parameters of the Standard Model. Quarks are confined inside the hadrons and are not observed as physical particles, therefore, quark masses cannot be measured directly but must be determined through their influence on hadronic properties.

Any quantitative statement about the value of a quark mass must refer to the particular theoretical framework that is used to define it. The quark masses for light quarks are often referred to as the current (bare) quark masses. Non-relativistic quark models use constituent (effective) quark masses, which are in the order of $\sim 350MeV$ for the u and d quarks. Constituent quark masses model the effects of dynamical chiral symmetry breaking and are not directly related to the quark mass parameters of the QCD Lagrangian. Constituent masses are only defined in the context of a particular hadronic model. For mass measurements in lattice gauge theory one computes a convenient and appropriate set of physical quantities, frequently chosen to be a set of hadronic masses, for a variety of input values of the quark masses; precise measurements are determined by the lattice spacing a, that is the distance between neighboring points of the lattice and quark masses. The true physical values of the quark masses are those which correctly reproduce the set of physical quantities being used for the calibration. In the particle data listings, quark masses have been obtained by using a wide variety of methods. Each method involves its own set of approximations and uncertainties. In most cases, the errors are an estimate of the size of neglected higher-order corrections or other uncertainties. It is also important to note that the quark mass values can be significantly different for different schemes. At low energy Quantum Chromodynamics (QCD) where both perturbation theory and asymptotic freedom are not possible, the collective interactions between valence and sea quarks become significant. The effects of virtual quarks and gluons in the sea of quarks are assigned to some particular quarks, which get surrounded by the dense cloud of virtual quarks and gluons. This cloud is a high energy barrier concealing the current quark at the core. This system is called constituent quark with an effective mass. The bare masses of u and d are so light that they cannot be straightforwardly calculated because relativistic effects have to be taken into account.

Form factors used to generate differential cross section of elastic electron-proton scattering ($dcsep$) were measured through various methods. One of which is by Rosenbluth Extraction Method which obtains them from the plot of the reduced cross section versus the square of the transfer momentum at several angles by determining the plot's slope and intercept, and performing linear regression with it [14]. With the world data [1,2,13,17,20], the form factor ratio is consistent to ~1.0 at very low energies. Another is by Polarization Transfer Method, wherein the form factor ratio is measured through polarization transfer where longitudinally polarized electron beam is scattered from an unpolarized proton target. For *ep*-scattering in the single-photon exchange, it was shown that the normal component of polarization vanishes and the transverse and longitudinal components satisfy certain conditions [15]. Without the need to measure the cross-sections, this gives the form factor ratio but which does not agree well Rosenbluth measurements. Form factor ratio from polarization transfer are well fit by 1-0.13($Q2$-0.04) [9]. The Super-Rosenbluth method of form factor extraction is associated with smaller angular-dependent corrections. And here, it is the protons that are scattered rather than electrons. Experimental data and results of elastic simulation from [14] used the Bosted global fit of previous Rosenbluth data from [4] due to the slight variation of Q^2 across the finite momentum and angle acceptance. P.E. Bosted ensured that form factors from the elastic simulation gave the closest corresponding final cross section. However, it is only valid at $0<Q^2<7GeV^2$. Another fitting procedure was presented in [22] and explained with details in [16]. For the proton, simultaneous fitting on form factors to the data were performed and the fit is a bounded polynomial *z*-expansion [12]. This global data fitting procedure is valid up to Q^2~$30GeV^2$.

Since the masses of *u* and *d* are not directly calculated, a device test can be formulated to estimate their masses via the measurement of the *dcs* generated from the form factors of elastic *ep*-scattering. The main objective of this study is to develop a technique in the mass estimation of *u*- and *d*-quarks with the proton mass as a parameter (and vice-versa) using the *dcsep* generated from theory and experiments. The investigation was mainly to determine the energy and *dcs* intersections of *dcsep*, *dcseu* and *dcsed* where the quarks assume the minimum/ maximum bare and effective masses by modifying the relativistic recoil factor using the proton mass at the low energy regime.

Methods

The generated *dcsep* data sets are compared to the modified *dcseq*, where *q* is either *u* and *d*, and their intersections were determined within the low energy regime. In the generation of *dcseq*, the quarks were assigned bare and effective (low energy) masses [10,18,19,21], separately. The relativistic recoil factor of *dcseq* of the spin-averaged *eq*-scattering is modified by using the mass of the proton in order to shift the distribution of *dcseq* closer to *dcsep* and thereby putting a possibility of finding an energy intersection with it. This energy intersection actually means the square of momentum transfer at which the two *dcs* meet. This materializes the proton mass a parameter in the determination of the quark mass estimates. The dipole proton form factors (dff) [11] and form factors derived from the fitting of experimental data, such as polarization transfer fitting (ptf) [14], Bosted global fitting (bgf) [4] and global data fit (gdf) [22]; and the average of all form factors (aff) were used to generate the data for *dcsep*. Using these form factors, the *dcsep*, *dcseu*, and *dcsed* were generated simultaneously in ROOT Data Analysis Framework [6]. The value for the anomalous magnetic moment of the proton used is ^=2.793 and the mass of proton is set at m p = 0.938272081±0.000000023 *GeV*. A total of 3000 events each for *dcsep*, *dcseu*, and *dcsed* were gathered at various scattering angles randomly selected within 0^o to 180^o. Then, they were averaged for each data point.

Analysis

The *dcsep* and *dcseq* are curves in the *dcs* versus Q^2 plot but there is no assurance of their intersection in at least one point unless alterations have to be implemented. So, the proton mass was required as a parameter in the quarks mass test. This can be done through modifying the relativistic recoil factor of *dcseq* using the mass of proton, instead. For a fixed scattering angle or considering all angles, the relativistic recoil factor is just a constant at a particular transfer momentum. Doing so does not alter the distribution, however, it shifts the *dcs* vertically. Considering this modification could make the possible attainment of an energy intersection. With the proton mass in the recoil factor, the existence of an energy intersection at low-energy is highly probable. The proton mass parameter becomes the link between the two *dcs* curves. It should be noted that polarization transfer fitting (ptf) diverges at $Q^2 \sim 7.7 GeV^2$ and Bosted global fit (bgf) is valid only at $Q^2 \sim 7 GeV^2$. Hence, it is important that the intersections should be below these valid transfer momentum limit. To give a general applicability of the technique, the values of *u*- and *d*-quark bare masses (minimum and maximum) and the effective masses were used. Effective quark masses are important since they dominate at low energy. Table 1 summarizes the bare and effective masses used as inputs to determine the energy and cross section intersections.

Table 1. Mass range for *u* and *d* in *MeV.*

Measurement schemes	u_{min}	u_{max}	d_{min}	d_{max}
MS scheme [19]	1.80	2.70	4.40	5.20
Lattice QCD [7]	1.87	2.15	4.63	4.95
Effective mass [10]	336		340	

Estimation of the masses of *u*- and *d*-quarks using the generated *dcsep* from form factors coming from different fitting models has to establish the energy at which *dcsep* and modified *dcseq* intersect with the globally accepted proton mass as a parameter. Once the energy intersections and their corresponding form factors are determined and established, then the *dcs* can be generated and from it the range of masses will be obtained. Oppositely, with an established energy intersection, a range of values of *u* and *d* quark masses can be plugged-in to the modified *dcseq* to obtain the accepted experimentally derived proton mass. Hence, a technique is developed for the mass estimation of *u*- and *d*-quarks using the generated *dcs*ep from form factors derived from experiments. Different experiments give different *dcs* corresponding to different form factors, whichever come first at a given energy, vesting a range of values for the quark masses as inputs to obtain the mass of the proton which is also determined in a multitude of ways.

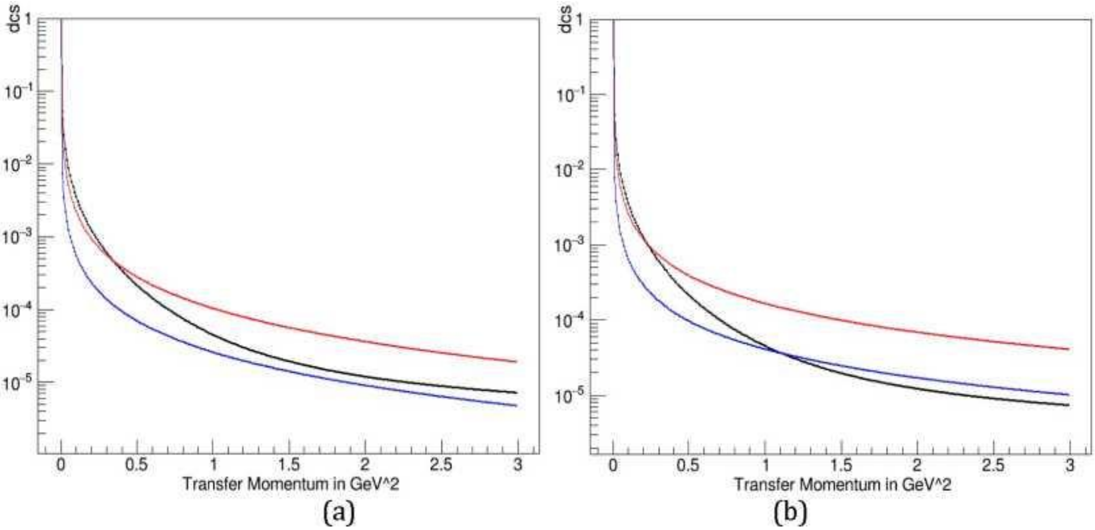

Figure 1. (a) The differential cross section (*dcs*) of the *eu*-scattering (red) and *ed*-scattering (blue) carrying effective masses are compared to that of *ep*-scattering (black) generated from the averaged form factors (aff). (b) The *dcseu* (red) and *dcsed* (blue) carrying effective masses are compared to *dcsep* (black) generated from aff with scattering angles ranging from 0^{o}-180^{o}. The intersections are pronounced here.

Results

The global data fit (gdf), one of the four form factor data fittings considered, does not agree well with the other three. This is due to the coefficient parameters of the curve. The limits of coefficient parameters are chosen only when actual data are available for analysis. For this study, it is assumed that the *dcs* data points generated from this fitting procedure are the deviants to the expected outcomes and they constitute 25% to the *dcs* generated from the average form factors (aff). The choice of coefficient parameters for gdf, to be within -0.214 to 0.214, is a compromised one and was based on the criteria that *dcsep* generated from gdf should produce an intersection with the modified *dcseq* not beyond $3GeV$ for both the modified *dcseu* and *dcsed*. It should be noted that gdf has a validity of up to $\sim 30GeV^2$, that is way beyond the limits of polarization transfer fitting (ptf) and Bosted global fit (bgf) which are valid only up to $\sim 7GeV^2$.

Comparisons of the generated *dcsep* between *dcseu* and *dcsed*, where quarks assume the bare masses, have large disparities and no intersections were observed. However, in Figure 1(a) where quarks have effective masses, intersection can exist between the curves, see Tables 2 and 3, for the summary of energy and *dcs* intersections. The extrapolated average energy intersection of *dcsep* from aff with *dcseu* is $346.00008MeV^2$.

It has no intersection with *dcsed* but they were closest at ~1954.54339MeV^2. Their corresponding extrapolated *dcs* are 4.58726x10-4 and ~1.1597x10^{-5} in the units of barn, respectively.

Table 2. Accessible low energy intersections of *dcsep* and *dcseq* in MeV^2 with bin size of 1 MeV^2 and where the quarks assume effective masses.

Quark mass	dff	pft	bgf	gdf	aff
^{u}effective	452.00004	485.00005	426.00001	2077.00001	346.00008
^{d}effective	1243.00018	1494.00005	1277.00001	281.00002	* ~1954.54339

*there is no intersection at this energy but the *dcs* are closest here.

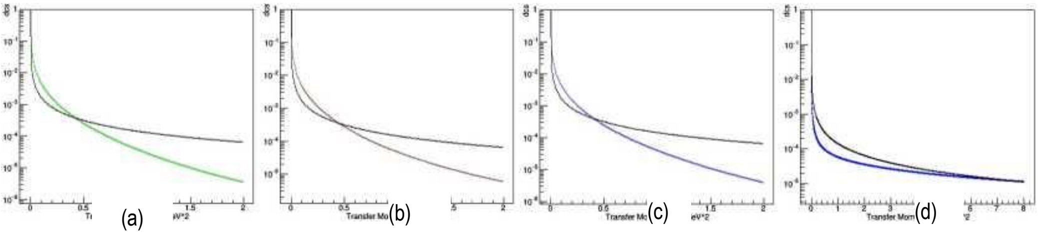

Table 3. Corresponding *dcs* (barns) intersections of *ep* and *eq*-scatterings in Table 2.

Quark mass	dff (x10^{-4})	pft (x10^{-4})	bgf (x10^{-4})	gdf (x10^{-4})	aff (x10^{-4})
^{u}effective	3.19285	2.89796	3.46249	0.34019	4.58726
^{d}effective	0.18923	0.14330	0.18169	1.52151	~0.11597

Figure 2. The *dcseu* (black) with minimum bare mass are compared to *dcsep* generated from (a) dff, (b) ptf, (c) bgf and (d) gdf form factor data sets.

Modifying the relativistic recoil factors caused the *dcseq* distributions to shift nearer to the *dcsep*, see Figure 1(b) for quarks carrying effective masses and Figure 2 for quarks carrying minimum bare masses. Due to the shift, intersections were observed. The energy intersections between *dcseu* and the generated *dcsep* from different form factor fitting were around the extrapolated value of 407MeV^2 to 450MeV^2 except for gdf which is beyond 7GeV^2. For *dcsed*, the energy intersections with *dcsep* occur beyond 1GeV^2 except for the ones generated by gdf which register around 183*MeV*2. In order to provide a range for the bare mass estimate of the quarks, their minimum and maximum values from MS scheme were used as inputs for this test.

Table 4. Low energy intersections of *dcsep* and *dcseq* in MeV^2 with bin size of 1 MeV^2.

Quark mass	dff	pft	bgf	gdf	aff
u bare mass min (ms)	428.00033	450.00017	407.00002	> 8000.00000	349.00016
u bare mass max (ms)	428.00032	450.00016	407.00003	> 8000.00000	349.00015
*d*bare mass min (ms)	1037.00000	1171.00025	1051.00015	183.00000	1272.00005
*d*bare mass max (ms)	1037.00001	1171.00027	1051.00016	183.00000	1272.00004
^{u}effective mass	324.00009	338.00005	297.00001	> 8000.00000	226.00013
^{d}effective mass	953.00003	1075.00014	962.00015	534.00004	1093.00004
^{u}bare mass min (lqcd)	428.00033	450.00017	407.00002	> 8000.00000	349.00016
^{u}bare mass max (lqcd)	428.00033	450.00017	407.00002	> 8000.00000	349.00016
^{d}bare mass min (lqcd)	1037.00001	1171.00026	1051.00016	183.00000	1272.00005
^{d}bare mass max (lqcd)	1037.00001	1171.00026	1051.00016	183.00000	1272.00004

Table 5. Corresponding *dcs* (barns) intersections of *ep* and *eq*-scatterings in Table 4.

Quark mass	dff (x10^{-4})	pft (x10^{-4})	bgf (x10^{-4})	gdf (x10^{-4})	aff (x10^{-4})
u bare mass min (ms)	3.61778	3.42626	3.82414	< 0.10935	4.51368
u bare mass max (ms)	3.61775	3.42622	3.82410	< 0.10935	4.51363
*d*bare mass min (ms)	0.33880	0.29505	0.33375	2.24898	0.26820
*d*bare mass max (ms)	0.33880	0.29504	0.33375	2.24889	0.26820
^{u}effective mass	6.56931	6.24876	7.29239	< 0.10959	10.07049

^{d}effective mass	0.43926	0.37751	0.43419	0.89979	0.36976
^{u}bare mass min (lqcd)	3.61778	3.42625	3.82413	< 0.10935	4.51367
^{u}bare mass max (lqcd)	3.61777	3.42624	3.82412	< 0.10935	4.51366
^{d}bare mass min (lqcd)	0.33880	0.29505	0.33375	2.24895	0.26820
^{d}bare mass max (lqcd)	0.33880	0.29504	0.33375	2.24892	0.26820

The extrapolated energy intersection of *dcsep* generated from aff with the modified *dcseu* carrying the minimum bare mass is 349.00016MeV^2 and this is greater than that of the modified *dcseu* carrying the maximum bare mass at 349.00015MeV^2; their respective extrapolated *dcs* intersections were at 4.51368x10^{-4} and 4.51363x10^{-4}. The extrapolated energy intersection of *dcsep* generated from aff with the modified *dcsed* carrying the minimum bare mass is 1272.00005MeV^2 and this is greater than that of the modified *dcsed* carrying the maximum bare mass at 1272.00004MeV^2; their respective extrapolated *dcs* intersections were both at 0.26820x10^{-4}. For quarks carrying effective masses, the extrapolated energy intersection of *dcsep* generated from aff with the modified *dcseu* is 226.00013MeV^2 and this is less than that of the modified *dcsed* at 1093.00004MeV^2 with respective *dcs* intersections at 9.83012x10^{-4} and 0.36743x10^{-4}. For quark masses calculated from Lattice QCD, the extrapolated energy intersection of *dcsep* generated from aff with the modified *dcseu* carrying the minimum and maximum bare masses were both at 349.00016MeV^2; their respective extrapolated *dcs* intersections were at 4.51367x10^{-4} and 4.51366x10^{-4}. The extrapolated energy intersection of *dcsep* generated from aff with the modified *dcsed* carrying the minimum bare mass is 1272.00005MeV^2 and this is greater than that of the modified *dcsed* carrying the maximum bare mass at 1272.00004MeV^2; their respective extrapolated *dcs* intersections were both at 0.26820x10^{-4}. All intersections are summarized in Tables 4 and 5.

Conclusions and Recommendations

Indeed, a device to estimate the range of quark masses for *u* and *d* via the elastic *ep*-scattering at low momentum transfer can be possible by generating *dcsep* data sets from form factors and comparing them to the modified *dcseq*, where *q* is either *u* and *d*, wherein the proton mass is a parameter. The theoretical dipole form factor and some form factor fitting models used in experiments such as the polarization transfer fitting, Bosted global fitting, global data fit and their average were used to generate the *dcsep*. The *dcsep*, *dcseu* and *dcsed* would have been independent of each other without the modification of the relativistic recoil factor since they do not possess a critical intersection that could be exploited. By using the mass of the proton into the relativistic recoil factor of the spin-averaged *eq*-scattering, the *dcseq* distribution was shifted closer *dcsep* and thereby increasing the possibility of intersection. Materializing the proton mass as a parameter, intersections were observed within the low energy regime by using the experimental quark masses as inputs. These intersections were summarized in Tables 2 through 5. Once established, these intersections can be used on experimental elastic *ep*-scattering data in order to estimate the masses of *u* and *d*, as well. Using most, if not all, data from actual experiments, a global range of *u* and *d* can be estimated. For the estimation of effective quark masses, both the intersections of the raw and modified *dcseq* have to be used simultaneously. One of the things to be done in making the intersections formidable would be incorporating additional form factor fitting models and to integrate the results of the actual experiments as they come. Generating the *dcs* with much smaller bin sizes is recommended to give a more precise measurement of the intersections. Also, choosing the optimum coefficient parameters for gdf could lead to a better result. Although the protons are measured in a multitude of ways, but more precise mass ranges of *u* and *d* are led by the precise measurement of its mass being the parameter of the test. The accuracy of the results can also be improved by generating even more events and considering more scattering angles. All of these recommendations, however, would entail much more computing power.

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