

# Using a Fast Elitist Non-Dominated Genetic Algorithm on Multi-Objective Programming for Quarterly Disaggregation of the Gross Domestic Product

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## Abstract

This research paper we use a fast elitist multiobjective genetic algorithm to solve the new approach that we propose to quarterly disaggregating of the Gross Domestic Product (GDP) by multiobjective programming. Thus, the quarterly disaggregation of the GDP is described as a quadratic multiobjective programming problem that generalizes Denton's proportional method. The proposed approach has the advantage reduce to one the number of optimization programs to be solved. Our proposed method can be applied to the national accounts of any country that has adopted the National Accounting System. The simulation results are compared to those obtained using Denton's proportional method and these results revealed the overall performance of the multiobjective programming approach for the quarterly disaggregation of GDP. Our approach is more suitable for taking into account the links between branches of national accounts, in terms of volumes and prices of products demanded during the production process. Also, it reduces forecast error and volatility of quarterly GDP. Besides, it is worth noting that our method is a usfull step for data processing such as chain-linked measures, overlap growth techniques, seasonal adjustment and calendar effects adjustment, in time series and econometrics analysis.

**Keywords:** Quarterly disaggregation, quarterly national accounts, optimization, quadratic program, multi-objective programming, genetic algorithm.

## 1. Introduction

In economics analysis, the main aggregate indicator resulting from the quarterly national accounts (QNA) is the quarterly Gross Domestic Product (GDP). Thus, the problem is that of quarterly GDP estimation for global cyclical economics analysis.

Generally, for having quarterly GDP, the economy is divided into several branches or sectors and the quarterly accounts estimation is done per branch defined through the System of National Accounting (SNA).

The procedures for compiling quarterly national accounts can be subdivided into direct and indirect procedures. The use of direct procedures requires the availability, at quarterly intervals, of the same data sources as those used to prepare the annual accounts, subject of course to the necessary simplifications. Direct procedures are often used in countries with sufficiently developed statistical systems. Indirect procedures is based on quarterly disaggregation of passed annual national accounts and extrapolation for current quarter by using mathematical or statistical techniques, with or without reference indicators. Indirect procedures are used in countries without sufficiently developed statistical systems as there is in sub-Saharan Africa. That are subdivided into two groups of methods: mathematical methods and econometric methods.

In the literature, mathematical methods and econometric methods are distinctly developed for the quarterly disaggregation of national accounts. A full presentation is given in the quarterly national accounts manual [11], [12]. The main difficulty in applying econometric methodologies is the need to have data available over a long period. Thus, the mathematical methods presents the advantage for applying them even with short data series [15]. Overall, mathematical methods based on numerical approach are adapted to less developed statistical systems [15] as in most of sub-Saharan Africa countries. The quarterly disaggregation according to mathematical methods preserves the infra-annual evolution of the quarterly indicator by ensuring consistency with the annual value of the national accounts [15]. Furthermore, the most practicable mathematical method using indicators is that developed by Denton (1971) which is presented in [11], [12] and the modified version done by Sjoberg (1982), Adriaan M. Bloem and al (2001), Cholette-Dagum (2006) can be seen in [12]. Denton method estimates quarterly accounts by proportional divisions based on the related quarterly indicator and also following a fairly smooth quarterly path based on certain criteria. Thus, the quarterly disaggregation of national accounts with Denton's method becomes a minimization problem with constraints. However, it is not evident to find a global composite quarterly indicator related to the GDP in order to quarterly disaggregate directly the GDP by this method.

As we can see, the traditional Denton's proportional method for the quarterly disaggregation of Gross Domestic Product (GDP) have many weaknesses related to the independent resolution of several optimization sub-problems resulting from the decomposition of GDP into branches of the economy according to the nomenclature defined for quarterly national accounts. The resolution of all these sub-problems one by one could be tedious due to the time required for individual processing. Therefore, this approach could lead to differences between the annual series of observed GDP and the GDP estimated from the quarterly accounts after aggregation due to simulation errors which occur at the level of the branches treated separately.

Based on the economy breakdown into several branches or sectors, this paper proposes a novel theoretical mathematical programming approach where the quarterly disaggregation of GDP is formalized as multiobjective optimization problem. Our paper solves the weaknesses of methods based on indirect procedures mainly Denton's proportional method for the GDP quarterly disaggregation. The overall quarterly disaggregation problem is presented using an indirect approach with indicators, leaning on Denton's proportional method. A Multi-Objective Programming for Temporal Disaggregation (MOPTD) model is therefore proposed (section 2) by generalizing Denton's proportional method. Moreover, we manage to build a non-trial hybrid algorithm based on an evolutionary fast elitist non-dominated genetic algorithm (NSGA-II) for solving the model (section 3).

After quarterly decomposition of national accounts by the MOPTD model, the derivated data can be used for some important data processing such as chain-linked measures, overlap growth techniques, seasonal adjustment and calendar effects adjustment, in order to get more consistency of quarterly time series for econometrics analysis. This paper proceeds neither chain-linked measures nor seasonal and calendar effects adjustment. The MOPTD model and the algorithm have been applied to Benin's real data of national accounts, breaking down the GDP into three sectors (primary, secondary and tertiary) (see section 4). The data were obtained from the National Institute of Statistics and Economic Analysis in short INSAE, of Benin.

## **2. Formulation of the Multi-Objective Programming for Temporal Disaggregation**

In this research, the quarterly disaggregation of branches accounts is based on an indirect approach with indicators. The basic idea of quarterly accounts is to "adapt" the indicators to the annual accounts, estimating the statistical relationship that links in the past, the annualised indicator to the corresponding account, and assuming that this relationship observed on annual data is also valid on quarterly data. Among the indirect methods, one of them retains our attention. This is the proportional method of Denton (1971) in [11, 12]. The choice is guided on the one hand by the fact that it minimizes the prediction error of the current year, so that the forecast annual growth rate is very close to the realization and on the other hand, by its ease of implementation because it is a method adapted to short time series. Several works in the literature have used Denton's basic formulation to propose modifications, as have Cholette and Dagum (1994) in [11] and Dagum and Cholette (2006) in [12]. It should therefore be noted that if the time series available on the annual national accounts are not long enough, the econometric approach could lead to inconsistent estimators. Therefore, this research is based on Denton's proportional method, which is also implemented in several countries.

First of all, it is necessary to make a description all things we need for the formulation of the problem.

### **Description of the entities, cardinalities and indexing of the problem**

We have the following entities :

M : the number of branches of national accounts, corresponding to the number of objective functions,

T : the number of years for national accounts observed,

The following indexes are chosen for the problem formalization:

$y \in \{1, 2, 3, \dots, T\}$ , year generic index,

$i \in \{1, 2, 3, 4\}$ , quarterly index,

$t \in \{1, 2, 3, \dots, 4T\}$ , generic index of quarters on the T years' period: the indexes  $i$  and  $y$  allowing to reference an observation are restated using an operator proposed by Raimi et al. [20]

$k \in \{1, 2, 3, \dots, M\}$ , generic branch index of the quarterly accounts nomenclature.

Description of the model variables

Data of the problem

Value of the annual account per branch: let  $Z_{k,y}$  be the (known) value added of the branch account , for the year  $y$ ;

quarterly indicator: let  $I_{k,t}$  be the value of the branch indicator  $k$ , for the quarter  $t = 1, 2, 3, \dots, 4T$ ;

$I_k = (I_{k,t})_{t=1,2,\dots,4T}$  : the vector of quarterly indicator related the branch  $k$  over the entire period ;

Inter-branch interaction: we note  $\bar{W}_{j,k}$  the interaction of the branch  $j$  on the branch  $k$ , considered as the average share of the branch  $k$  demand of product coming from the branch  $j$  :  $\bar{W}_{j,k} = 1$  if  $j = k$  and  $0 \leq \bar{W}_{j,k} < 1$  if  $j \neq k$ .

Target variables

Value added per branch: let  $X_{k,t}$  be the value of national account for branch  $k$  at quarter  $t = 1, 2, 3, \dots, 4T$

The vector of national account value for branch  $k$  over the entire period is noted:

$$X_k = (X_{k,t})_{t=1,2,\dots,4T}, X_k \in \mathbb{R}^{4T}; \text{ for } k = 1, \dots, M;$$

The vector of quarterly national accounts value for all branches over the entire period is noted:  $X = (X_1, X_2, X_3, \dots, X_M)$ ,  $X \in \mathbb{R}^{4T \times M}$  ;

Quarterly Gross Domestic Product for the quarter  $t$  :  $GDP_t$  .

### ***Relationships between variables and parameters***

The target quarterly national accounts are related to the indicators (input data) through proportionality relationships.

The proportionality relationship between estimated quarterly values added and quarterly related indicators is as follow:

$$\frac{X_{k,t}}{I_{k,t}} = \frac{X_{k,t-1}}{I_{k,t-1}} + u_{k,t} ; \quad \forall k = 1, 2, \dots, M; \quad \forall t = 2, \dots, 4T \quad (2)$$

$u_{k,t}$  is the random term not explained for the quarter  $t$ .

The first difference in the BI ratio results in the quarterly residues

$$u_{k,t} = \frac{X_{k,t}}{I_{k,t}} - \frac{X_{k,t-1}}{I_{k,t-1}} ; \quad \forall k = 1, 2, \dots, N; \quad \forall t = 2, \dots, 4T \quad (3)$$

The sum of the squares of the quarterly residues that must be minimal is as followed

$$\sum_{t=2}^{4T} \left( \frac{X_{k,t}}{I_{k,t}} - \frac{X_{k,t-1}}{I_{k,t-1}} \right)^2, \text{ for } k = 1, 2, \dots, M \quad (4)$$

The quarterly Gross Domestic Product is expressed by the relationship:

$$GDP_t = \sum_{k=1}^M X_{k,t} ; \quad \text{for } t = 1, 2, \dots, 4T \quad (5)$$

Annual Gross Domestic Product for the year is obtained by aggregation over the four quarters

$$GDP_y = \sum_{t=4y-3}^{4y} GDP_t ; \quad \text{for } y = 1, 2, \dots, T \quad (6)$$

### ***Objective functions***

For the analysis with a multiobjective approach, the interaction between all branches is taken into account. In addition, it is assumed that there is an influenced additivity between sectoral residues in the determination of quarterly national accounts. Thus, the objective functions are given by :

$$f_k(X_1, X_2, \dots, X_M) = \sum_{j=1}^M \sum_{t=2}^{4T} \bar{W}_{j,k} \left( \frac{X_{j,t}}{I_{j,t}} - \frac{X_{j,t-1}}{I_{j,t-1}} \right)^2 ; \quad \text{for } k = 1, 2, \dots, M \quad (7)$$

### **Constraints of the model**

In each branch, the sum of the quarterly accounts estimated for the four quarters of a year is equal to the annual account (value added) of the branch for that year:

$$\sum_{t=4y-3}^{4y} X_{k,t} = Z_{k,y} ; \quad \forall y = 1, 2, \dots, T; k = 1, 2, \dots, M \quad (8)$$

At each year, the total of the quarterly values added towards all branches is equal to the GDP:

$$\sum_{k=1}^M \sum_{t=4y-3}^{4y} X_{k,t} = \sum_{k=1}^M Z_{k,y} ; \quad \forall y = 1, 2, \dots, T \quad (9)$$

For the variables sign, since the values of the branches annual accounts are positive (if not they can be made positive), it is also assumed that the corresponding quarterly values are positive:

$$X_{k,t} \geq 0 ; \quad \text{for } t = 1, 2, \dots, 4T \text{ and } k = 1, 2, \dots, M \quad (10)$$

With regard to maximum values, for a given year, the quarterly values of the accounts may not exceed the annual sectoral value:

$$X_{k,t} \leq Z_{k,y} ; \quad \forall t = 1, 2, \dots, 4T ; k = 1, 2, \dots, M ; \forall y = 1, 2, \dots, T \quad (11)$$

Using remark 1 (a), an additional constraint, relating to the weighted averages of the BI ratios, is added to Denton's traditional equilibrium ratios. It result in :

$$\sum_{t=4y-3}^{4y} \frac{X_{k,t}}{I_{k,t}} \eta_{k,t} = \frac{Z_{k,y}}{\sum_{t=4y-3}^{4y} I_{k,t}} ; \quad \forall y = 1, 2, \dots, T, \quad k = 1, 2, \dots, M ,$$

The weights are given by : (12)

$$\eta_{k,t} = \frac{I_{k,t}}{\sum_{r=4y-3}^{4y} I_{k,r}} ; \quad k = 1, 2, \dots, M$$

**Remark 2**

If the constraints translated by the relationships (8) and (10) are satisfied then the constraint translated by the relationship (11) is satisfied. Moreover, the relationship (8) implies the relationship (9). As a result, constraints (9) and (11) are not taken into account in the theoretical model but will be taken into account in the simulation algorithms in order to reinforce the constraints for minimizing the value of the objective functions.

**Definition 3 (MOPTD)**

By grouping together all the elementary objective functions identified by the relationship (7) for all branches, and the constraints identified by the relationship (8), (10) and (12), the quarterly disaggregation of GDP appears as a problem formulated in the form of multiobjective programming. Thus, the multiobjective programming temporal disaggregation model proposed in this research is as follows:

$$\begin{aligned} & \min_X \{f_1(X), f_2(X), \dots, f_k(X), \dots, f_M(X)\} && \text{(MOP)} \\ & \text{subject to.} \\ & X = (X_1, X_2, X_3, \dots, X_M) \\ & X_k = (X_{k,t})_t ; t = 1, 2, \dots, 4T ; \text{ pour tout } k = 1, 2, \dots, M \\ & -X_{k,t} \leq 0 ; \quad \forall t = 1, 2, \dots, 4T ; k = 1, 2, \dots, M && \text{(i)} \\ & \sum_{t=4y-3}^{4y} X_{k,t} - Z_{k,y} = 0 ; \quad \forall y = 1, 2, \dots, T ; k = 1, 2, \dots, N && \text{(ii)} \\ & \sum_{t=4y-3}^{4y} \frac{X_{k,t}}{I_{k,t}} \eta_{k,t} = \frac{Z_{k,y}}{\sum_{t=4y-3}^{4y} I_{k,t}} ; \quad k = 1, 2, \dots, M, \quad \forall y = 1, 2, \dots, T && \text{(iii)} \\ & \text{where } GDP_t = \sum_{k=1}^N X_{k,t} ; t = 1, 2, \dots, 4T \end{aligned}$$

As it can be seen, this problem is multiobjective quadratic programming.

To reconcile the estimated quarterly national accounts with the true annual observed values, an adjustment is made to the estimated quarterly values. The adjusted value  $X_{k,t}^{Adj}$  of the estimated national accounts for quarter  $t$  for branch (sector)  $k$  is given by:

$$X_{k,t}^{Adj} = \hat{X}_{k,t} - \eta_{k,t} \times Abs \left( \sum_{t=4y-3}^{4y} \hat{X}_{k,t} - Z_{k,y} \right) \times sign \left( \sum_{t=4y-3}^{4y} \hat{X}_{k,t} - Z_{k,y} \right) \quad (13)$$

$\forall y = 1, 2, \dots, T$

The weights are given by:  $\eta_{k,t} = \frac{I_{k,t}}{\sum_{t=4y-3}^{4y} I_{k,t}}$

$\hat{X}_{k,t}$  is the estimated value of the account for the quarter and  $Z_{k,y}$  the value for the year  $y$

$Abs(x)$  is the absolute value of  $x : Abs(x) = \max\{-x, x\}$

$sign(x) = 1$  if  $x > 0$  and  $sign(x) = -1$  if  $x < 0$ .

The interest of the quarterly national accounts lies in the fact that in a current year when the annual accounts are not yet available, the quarterly accounts for that year can be estimated by extrapolation from the values of the observed quarterly indicators. Thus, starting from Denton's basic extrapolation method and that presented by Di Fonzo, T. and Marco, M. [9] in their formula (5), the relationship presented by Marco [18] through scenario 3, is adapted to the quarterly accounts by considering the values of the BI ratios of the last nine previous quarters. Assuming that  $m$  is the last year of observed annual national accounts, the extrapolated value added of the branch (sector)  $k$  for the quarter  $(4m + r)$  is given by :

$$X_{k,4m+r}^{Adj} = I_{k,4m+r} \left[ 0.5 \times \frac{X_{k,4m+r-1}^{Adj}}{I_{k,4m+r-1}} + 0.5 \times \left( \frac{1}{9} \sum_{t=(4m+r-1)-9}^{(4m+r-1)} \frac{X_{k,t}^{Adj}}{I_{k,t}} \right) \right] \quad (14)$$

where

$X_{k,t}^{Adj}$  is the adjusted interpolation value added of the branch  $k$  for the quarter  $t$   
 $r$  is the rank of the quarter of year  $m + 1$  for which the extrapolation is made,  
 $r = 1, 2, 3, 4$ .

**Proposition 1**

Under the assumption of a total absence of interactions between branches, the program (MOP) is reduced to the Denton proportional method applied to each of the  $M$  branches of national accounts.

**Proof**

Let suppose that there is no interaction between the  $M$  branches of national accounts.

So we have  $\bar{W}_{j,k} = 1$  if  $j = k$  et  $\bar{W}_{j,k} = 0$  if  $j \neq k$  ;  $k = 1, 2, 3, \dots, M$

Consequently,  $f_k$  becomes

$$f_k(X) = \sum_{t=2}^{4T} \left( \frac{X_{k,t}}{I_{k,t}} - \frac{X_{k,t-1}}{I_{k,t-1}} \right)^2 ; \quad \forall k = 1, 2, \dots, M \text{ with } X = (X_1, X_2, X_3, \dots, X_M),$$

$f_k(X)$  therefore depends only on  $X_k$  ,  $\forall k = 1, 2, \dots, M$  . The problem (MOP) can therefore be written as follows:

$$\begin{aligned} & \text{Min}_{X_1, X_2, X_3, \dots, X_M} \{ (f_1(X_1), f_2(X_2), \dots, f_k(X_k), \dots, f_M(X_M)) \} && \text{(MOP)'} \\ & \text{subject to} \end{aligned}$$



$$\left\{ \begin{array}{l} X = (X_1, \quad X_2, \quad X_3, \dots, X_M) \\ X_k = (X_{k,t})_t ; \quad t = 1, 2, \dots, 4T ; \text{ for all } k = 1, 2, \dots, M \\ -X_{k,t} \leq 0 ; \quad \forall t = 1, 2, \dots, 4T ; k = 1, 2, \dots, M \\ \\ \sum_{t=4y-3}^{4y} X_{k,t} - Z_{k,y} = 0 ; \quad \forall y = 1, 2, \dots, T ; \quad k = 1, 2, \dots, M \\ \\ \sum_{t=4y-3}^{4y} \frac{X_{k,t}}{I_{k,t}} w_{k,t} = \frac{Z_{k,y}}{\sum_{t=4y-3}^{4y} I_{k,t}} ; \quad k = 1, 2, \dots, M, \quad \forall y = 1, 2, \dots, T \end{array} \right.$$

By relaxing the constraint  $\sum_{t=4y-3}^{4y} \frac{X_{k,t}}{I_{k,t}} w_{k,t} = \frac{Z_{k,y}}{\sum_{t=4y-3}^{4y} I_{k,t}} ; \quad k = 1, 2, \dots, M, \quad \forall y = 1, 2, \dots, T,$

we get the following new program:

$$\begin{array}{l} \text{Min}_{X_1, X_2, X_3, \dots, X_M} \{ (f_1(X_1), f_2(X_2), \dots, f_k(X_k), \dots, f_M(X_M)) \} \quad \text{(MOP)}'' \\ \text{subject to} \\ \left\{ \begin{array}{l} X = (X_1, \quad X_2, \quad X_3, \dots, X_M) \\ X_k = (X_{k,t})_t ; \quad t = 1, 2, \dots, 4T ; \text{ for all } k = 1, 2, \dots, M \\ -X_{k,t} \leq 0 ; \quad \forall t = 1, 2, \dots, 4T ; k = 1, 2, \dots, M \\ \\ \sum_{t=4y-3}^{4y} X_{k,t} - Z_{k,y} = 0 ; \quad \forall y = 1, 2, \dots, T ; \quad k = 1, 2, \dots, M \end{array} \right. \end{array}$$

Since the objective functions have independent arguments, it is possible to optimize the  $f_k$  separately to determine the  $X_k = (X_{k,t})_t$  for  $k = 1, 2, \dots, M$ , so that we can have the  $GDP_t$  by summing the  $X_{k,t}$ , using relationship (5).

This means solving all the elementary optimization program  $(EOP)_k$  :

$$\begin{array}{l} \text{Min}_{X_k} \{ f_k(X_k) \} \quad \text{(EOP)}_k \\ \text{subject to} \\ -X_{k,t} \leq 0 ; \quad \forall t = 1, 2, \dots, 4T ; \\ \sum_{t=4y-3}^{4y} X_{k,t} - Z_{k,y} = 0 ; \quad \forall y = 1, 2, \dots, T \end{array}$$

It can be seen that the program  $(EOP)_k$  is similar to Denton 's program presented in [11], illustrating the Denton proportional method applied to branch  $k = 1, 2, \dots, M$  of national accounts.

Thus, the program (MOP) is reduced to Denton's proportional method applied successively to the M branches to deduct quarterly GDP from the previous relationship (5). ■

### **Remark 3**

From the results of proposition 1, it can be deduced that the proposed MOPTD model is a generalization of Denton's proportional method.

### 3. Resolution approach and algorithm design

The problem is analysed on the basis of a hypothesis of cooperation between the objective functions. On this basis, Pareto optimal solutions are prospected in solving the problem. The following definition provides an understanding of the Pareto optimality concept.

#### **3.1. Preliminary definition ([19])**

Let's consider the vector function  $F(x) = (f_1(x), f_2(x), \dots, f_Q(x))$  and  $U$  the constrained space (space of feasible solutions) of the problem (MOPP). It should be noted  $F(U)$  the value space of the objective functions. We define on  $F(U)$  a partial relationship. Let  $K$  be any blunt cone such that  $K \subset \mathbb{R}^Q$ . Let's consider the binary relationship  $\preceq_K$  indexed by  $K$  defined as follow:

$$a \preceq_K b \Leftrightarrow (b - a) \in K.$$

Since it is not possible to find a solution that simultaneously optimizes all objective functions in the case of a multiobjective program, the notion of dominance in the sense of Pareto is used.

(i)- Pareto-dominance: for two feasible decision vectors  $x$  et  $y$ , we say that  $x$  dominates  $y$  in the sense of Pareto and we note  $(x, F(x)) \preceq_K (y, F(y))$ , if and only if for all  $q \in \{1, 2, 3, \dots, Q\}$ ,  $f_q(x) \leq f_q(y)$  and  $\exists q_0 \in \{1, 2, 3, \dots, Q\}$  such that  $f_{q_0}(x) < f_{q_0}(y)$ .

(ii)- Pareto optimal solution: a solution  $x \in \mathbb{R}^N$  is called Pareto optimal in  $\mathbb{R}^N$  if and only if, there is no vector  $y \in \mathbb{R}^N$  which dominates  $x$ .

(iii)- The Pareto optimal set is defined as the set of all Pareto optimal solutions.

(iv)- The Pareto optimal front is defined as the set of all objective function values corresponding to all solutions in the Pareto optimal set.

The resolution algorithms of the traditional quarterly disaggregation methods are presented in [1], [2]. In this paper, the simulation method for the model resolution is based on the NSGA-II algorithm developed in the literature for multiobjective optimization [3], [4], [8], [21].

### 3.2. A brief overview of the NSGA-II algorithm

The NSGA algorithm called "Non-dominant Sorting Genetic Algorithm"[10, 20], after several years of implementation, has become ineffective in ranking individuals [14] especially for large problems treated. To correct this deficiency, elitism was introduced into the basic algorithm [8] in order to preserve the best solutions from generation to generation [14]. Thus, the improved version of the NSGA algorithm, called NSGA-II, classify the individuals into many levels [6], this classification does not require necessary the choice of sharing parameter [4]. The NSGA-II algorithm is illustrated by the pseudo code given in algorithm 1. It should be noted that elitism in the NSGA-II algorithm contributes to accelerating the rate of convergence and the overall performance of the genetic algorithm incorporated in it.

#### *Algorithm 1 (pseudo-code NSGA-II) [8, 14]*

- While (total number of iterations not completed)
- Generation of the initial population
- Repeat
- While (Population is not classified) do
- Evaluation of all borders
- Normalization of areas of constraint violation
- Search for undominated individuals
- Replacement of individuals
- End while
- Selection
- Crossover
- Polynomial mutation
- Until (Criteria for shutdown achieved)
- Recombination of optimal Pareto solutions
- End while

### 3.3. The proposed MOPTD algorithm designing

The NSGA-II adapted to the problem of multi-target optimization for the quarterly disaggregation of GDP resulted in a fast elitist multiobjective programming algorithm for temporary disaggregation (MOPTD-NSGA-II). The choice of the NSGA-II algorithm as the central nucleus is justified by the fact that elitist algorithms allow better results to be obtained on multiobjective problems [5]. The pseudo code of the MOPTD -NSGA-II algorithm is given in Algorithm 5. The interpolation of the accounts by the model takes into account the reconciliation of the quarterly accounts with the true annual values observed. This reconciliation is done by an adjustment procedure of which the pseudo-code is given in algorithm 3. In addition, the MOPTD model allows quarterly accounts to be extrapolated if necessary using the extrapolation procedure of which the pseudo code is given in algorithm 4.

In the NSGA-II algorithm, the initial population is generated from the bounded values (minimum and maximum) of the target variables. Since the problem (MOP) is relatively of large size, to ensure rapid convergence of the algorithm towards Pareto optimal solutions, the solution search domain has been narrowed using the annual data provided on account variables. Thus, the variable bounded values were generated from the annual values of the accounts by releasing the constraints (i) of the problem (MOP). The pseudo code of the limit value calculation procedure is given in algorithm 2.

As it can be seen, the NSGA-II procedure introduces a hazard when generating the reference population. Since the search for Pareto optimal solutions is based on this reference population and the Pareto optimal solution does not strictly achieve the minimum of each objective function, an adjustment procedure has been completed in order to reconcile the estimated quarterly values with the true observed annual values. The relationship (13) presented in the theoretical description is used. The pseudo-code of this adjustment procedure is given in algorithm 3.

**Algorithm 2 (pseudo code of the Procedure Boundary\_values\_Calculation (abX, ny))**

- BEGIN (Boundary\_values\_Calculation)
- \*/Initialization of the values of the annual accounts (all branches/sectors))
- For k = 1 : M
- For y = 1 : lyear
- abX(k,y)  $\leftarrow$  annual values
- end for
- End for
- \*/Calculation of boundary values for quarterly accounts
- For k = 1 : M
- qbX\_max (k)  $\leftarrow$  [] \*/ empty box
- qbX\_min (k)  $\leftarrow$  [] \*/ empty box
- For y = 1 : ny
- qbXmax  $\leftarrow$  []
- qbXmin  $\leftarrow$  []
- For t = 4y-3 : 4y
- qbX\_max (k,t)  $\leftarrow$  abX(k,y)/4+[SD (abX(k))]/(lyear-fyear+1)
- qbX\_min (k,t)  $\leftarrow$  abX(k,y)/4-[SD (abX(k))]/(lyear-fyear+1)
- qbXmax  $\leftarrow$  [qbXmax qbX\_max (k,t) ]
- qbXmin  $\leftarrow$  [qbXmin qbX\_min (k,t) ]
- t  $\leftarrow$  t+1
- End for
- qbX\_max (k)  $\leftarrow$  [qbX\_max (k) qbXmax]
- qbX\_min (k)  $\leftarrow$  [qbX\_min (k) qbXmin]
- y  $\leftarrow$  y+1

- End for
- $qbX\_M(k) \leftarrow qbX\_max(k, 1:nq)$
- $qbX\_m(k) \leftarrow qbX\_min(k, 1:nq)$
- $k \leftarrow k+1$
- End for
- $tXmax \leftarrow [qbX\_M(1) \quad qbX\_M(2) \quad qbX\_M(3) \quad \dots \quad qbX\_M(M)]$
- $tXmin \leftarrow [qbX\_m(1) \quad qbX\_m(2) \quad qbX\_m(3) \quad \dots \quad qbX\_m(M)]$
- END (Boundary\_values\_Calculation)

***Algorithm 3 (pseudo code of the procedure Adjustment (qX,ny,nq))***

- BEGIN (adjustment procedure)
- Initialization
- vector of the abX annual accounts
- vector of quarterly accounts qX
- vector of quarterly indicators qbl
- For  $k = 1 : M$
- $qX(k)\_Adj \leftarrow [ ]$
- For  $cy = 1 : ny$
- $fcount \leftarrow 4*cy-3;$
- $lcount \leftarrow 4*cy;$
- $qX\_a(k, cy) \leftarrow [ ]$
- For  $j = fcount : lcount$
- $w(k,j) \leftarrow qbl(k,j) / \text{sum}(qbl(k,j), j, fcount : 1: lcount)$
- $c(1, cy) \leftarrow \text{sum}(qX(k,j), j, fcount : 1: lcount) - abX(k, cy)$
- If  $c(1, cy) \geq 0$
- $qx\_a(k, j) = qX(k, j) - w(k,j) * \text{abs}(c(1, cy))$
- Else
- $qx\_a(k, j) = qX(k, j) + w(k,j) * \text{abs}(c(1, cy))$
- End if
- $qX\_a(k, cy) \leftarrow [qx\_a(k, cy) \quad qx\_a(k, j)]$
- $j \leftarrow j+1$
- End for
- $qX(k)\_Adj \leftarrow [qX(k)\_Adj \quad qX\_a(k, cy)]$
- $cy \leftarrow cy+1$
- End for
- $k \leftarrow k+1$
- End for
- END (adjustment procedure)

The extrapolation of quarterly accounts for future quarters is based on the procedure described by the relationship (14) presented in the theoretical description. The pseudo code of the extrapolation procedure is given in algorithm 4.

**Algorithm 4 (pseudo code of the procedure Extrapolation (qX\_entry, qbl\_new, nq ,s))**

- BEGIN (extrapolation procedure)
- $s \leftarrow 4 \times (\text{lyear} - \text{wyear})$
- While ( $s > 0$ ) do
- Call for data on quarterly indicators  $I(1:nq+s)$
- For  $k = 1 : M$
- Colling of the vector  $I(k, 1:nq+s)$
- $X(k, 1:nq) \leftarrow qX\_adjus(k, 1:nq)$
- $X(k) \leftarrow X(k, 1:nq)$
- $X(k, nq+1:nq+s) \leftarrow [ ]$
- For  $r = 1 : s$
- $X(k, nq+r) \leftarrow I(k, nq+r) * ( 0.5 * X(k, nq+r-1) / I(k, nq+r-1)$   
 $\quad \circ + 0.5 * \text{sum}(X(k, t) / I(k, t), (nq+r-1):1: (nq+r-1)-9)/9)$
- $X(k, nq+1:nq+s) \leftarrow [ X(k, nq+1:nq+s) \quad X(k, nq+r) ]$
- $r \leftarrow r+1$
- End for
- $X(k) \leftarrow [X(k) \quad X(k, nq+1:nq+s)]$
- $k \leftarrow k+1$
- End for
- Recovery and backup of the matrix  $[ X(1) \quad X(2) \quad X(3) \quad \dots \quad X(M) ]$
- End while
- END (extrapolation procedure)

Finally, the pseudo code of the complete algorithm (MOPTD-NSGA-II) is given in algorithm 5 which is as follows :

**Algorithm 5 (main pseudo code of MOPTD-NSGA-II)**

- BEGIN (main Algorithm)
- \*/Initialization time parameters
- fyear     \*/ first year of the observation period of the annual accounts
- lyear     \*/ last year of the observation period of the annual accounts
- wyear     \*/ last year considered for interpolation
- \*/ Calculation of all other associated parameters
- $wy \leftarrow (wyear - fyear + 1)$      \*/ number of work years for interpolation
- $wq \leftarrow 4 * wy$      \*/ number of quarters for the calculation period
- $s \leftarrow 4 \times (\text{lyear} - \text{wyear})$      \*/ number of quarters for extrapolation
- If  $\text{lyear} > \text{wyear}$  then

- $ny \leftarrow wy;$                    \*/ number of years for interpolation
- $nq \leftarrow wq;$                    \*/ number of quarters for calculations
- else
- $ny \leftarrow (\text{year} - \text{fyear} + 1)$    \*/ number of years for interpolation
- $nq \leftarrow 4 * ny$                    \*/ number of quarters for calculations
- End if
- Boundary\_values\_Calculation (abX, ny)   \*/ Execution of the Procedure
- \*/ Interpolation of accounts on quarterly indicators
- Begin (interpolation)

- Initialization of the other NSGA-II input parameters  
 $M \leftarrow$  number of objective functions  
 $p1 \leftarrow$  number of constraints  
 $V \leftarrow nq * M$    \*/calculates the number of variables  
 $\text{Pop\_size} \leftarrow$  give the size of the population  
 $\text{run} \leftarrow$  the number iteration  
 $\text{gen\_max} \leftarrow$  give the maximum number of generations  
- Calling up the problem (objective functions and constraints))

- Begin (NSGA)
- Execution of the NSGA-II algorithm
- End (NSGA)
- \*/ Recovery of results provided by NSGA
- If  $\text{run} == 1$
- $qX \leftarrow [\text{new\_pop}(:, 1:V)]$                    \*/ Pareto optimal solution
- Else
- $qX \leftarrow [\text{pareto\_rank1}(1:V)]$                    \*/ Pareto optimal solution
- End if
- \*/ Adjustment of quarterly accounts
- For  $k = 1 : M$
- Adjustment (qX(k), ny, nq)   \*/ Execution of the procedure
- $qX(k)\_Adj \leftarrow$  Adjustment (qX(k), ny, nq)
- $k \leftarrow k + 1$
- End for
- End (interpolation)
- \*/ Extrapolation of accounts to the quarters of the following year
- If ( $s > 0$ )

- Calling up indicator values  $qbl(1:nq+s)$
- Begin (extrapolation)
- For  $k = 1 : M$
- Extrapolation ( $qX(k)_{Adj}$ ,  $qbl(k)$ ,  $nq$ ,  $s$ ) \*/ Execution of the procedure
- $qX1\_fwd(k) \leftarrow$  Extrapolation ( $qX(k)_{Adj}$ ,  $qbl(k)$ ,  $nq$ ,  $s$ )
- $k \leftarrow k+1$
- End for
- End (extrapolation)
- En if
- END (main algorithm)

#### 4. Application to Benin's national accounts

Simulations are carried out using the databases of the National Institute of Statistics and Economic Analysis (INSAE). The data were subject to prior statistical processing before proceeding to the actual simulations. The GDP is decomposed into three sectors. The results obtained with the proposed model were compared with those obtained using Denton's proportional method applied separately to each of the three sectors.

##### 4.1. Statistical data and processing

This section presents the data sources we used and the processing carried out on data related to national accounts and related indicators.

The databases available at the National Institute of Statistics and Economic Analysis (INSAE) reveals that Benin has the series of annual national accounts (ANA) from 1999 to 2015, compiled according to the System of National Accounting (SNA) 93 with 2007 as the base year, as well as various (quarterly) indicators on economic activity. These data were therefore used for the simulation exercise. It should be noted that work undertaken by the INSAE national accounts department to move from SNA 93 to SNA 2008 [10], with a base year change to 2015, has not yet been completed to make available the new series on the definite accounts over the simulation period.

In the context of this research, the quarterly GDP obtained through the quarterly disaggregation of the values added. Thus, the proposed approach using three indicators is different from that of Abdelwahed Trabelsi and Leila Hedhili [1] for which GDP is quarterly in aggregate form with only one indicator, the industrial production index.

Thus, the resolution approach adopted is that relating to the research for efficient points (Pareto optimum) after problem reduction (*MOP*) with three objectives.

##### 4.2. Our method results and comparison with Denton's proportional method

Then, several tests were performed on the key parameters of the algorithm in order to retain values that reduce the convergence time of the algorithm. Simulation results and statistics on the quality of the model are presented below. In the literature, some



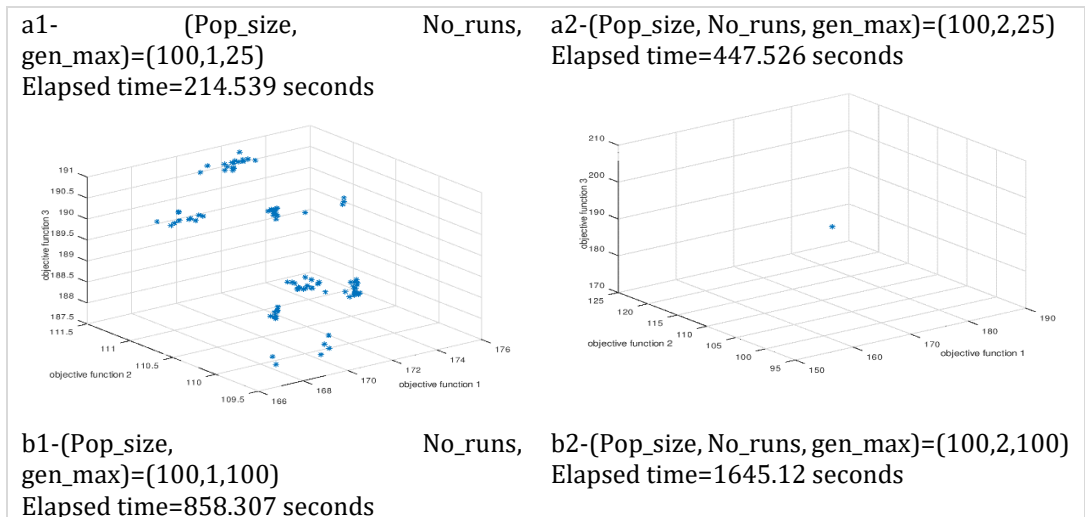
softwares are adopted for the quarterly disaggregation of national accounts using traditional methods. As part of this research, all simulations are carried out using the OCTAVE software.

### ***Pareto optimality front***

The NSGA-II algorithm uses the following key parameters as input parameters: population size (`pop_size`), iteration number (`no_run`) and maximum number of generations (`gen_max`) beyond which the algorithm stops. The choice of the values of these parameters depends on the size of the problem. In the literature, problems are tested with the NSGA-II algorithm for minimum required values and set at 20 for `pop_size` and 5 for `gen_max` [4]. Sometimes, large values can be set: up to 200 for `pop_size`, 10 for `no_run` and 1000 for `gen_max` in the case of problems with two objectives [8].

Based on these findings, several values were tested for the parameters `pop_size` as in [14], `no_run`, and `gen_max`, in this research. These tests have been illustrated by different representations of the Pareto front. The Pareto front obtained for the main values of (`pop_size`, `no_run`, `gen_max`) are presented in Figure 1. The Pareto optimal points are represented in blue.

The analysis of the results shows that if `no_run`=1, the simulation gives several Pareto optimal points but the number of points increases with the population size (`pop_size`) and this regardless of the `gen_max` value, this situation is illustrated by the panels (a1) (b1), (c1) and (c2). It was used in the following `pop_size`=100 simulations, as adopted in [6] and [21].



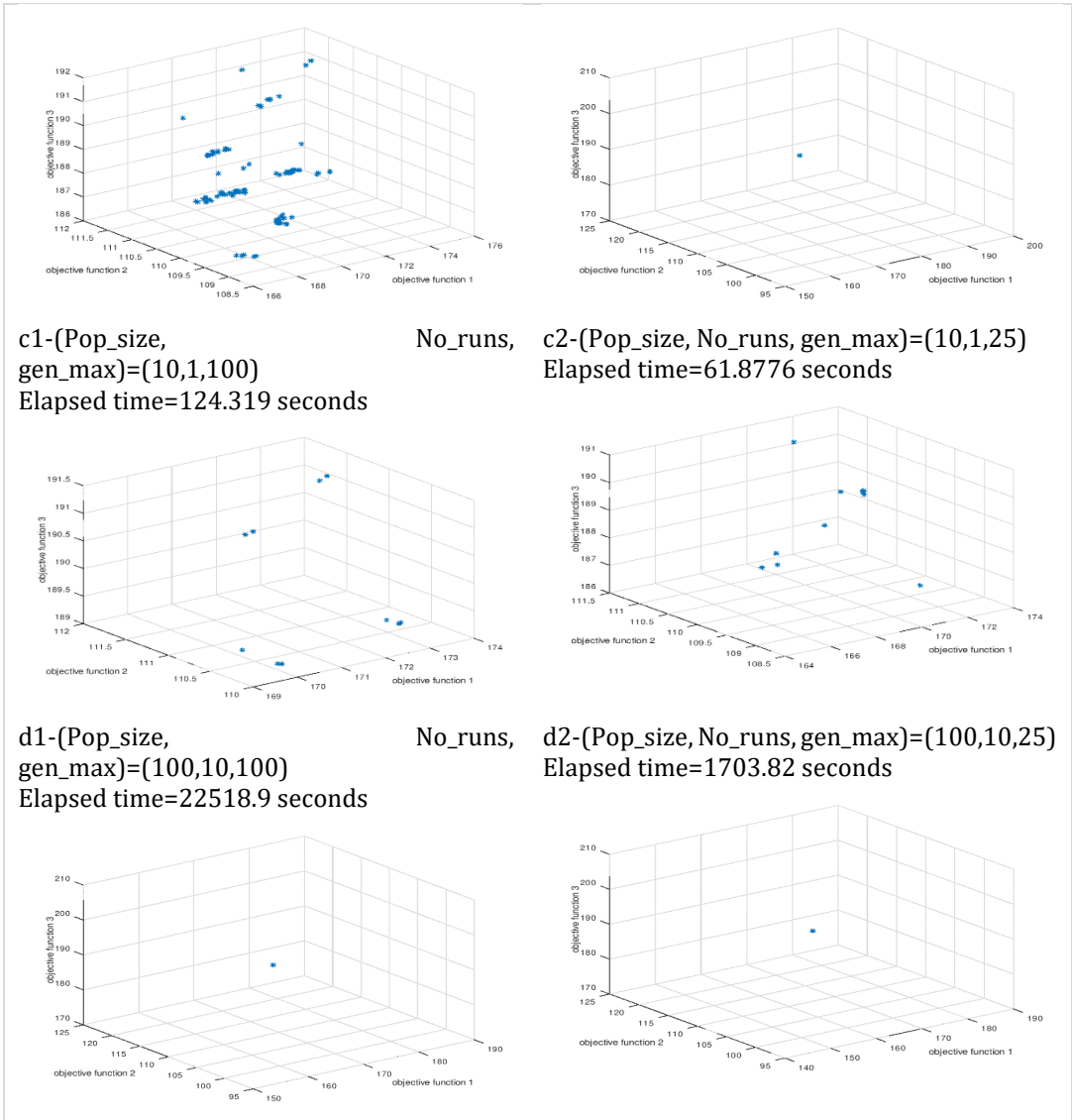


Figure 1 : Optimal Pareto boundary for different simulation parameters

Source: INSAE-data base, Author's simulation

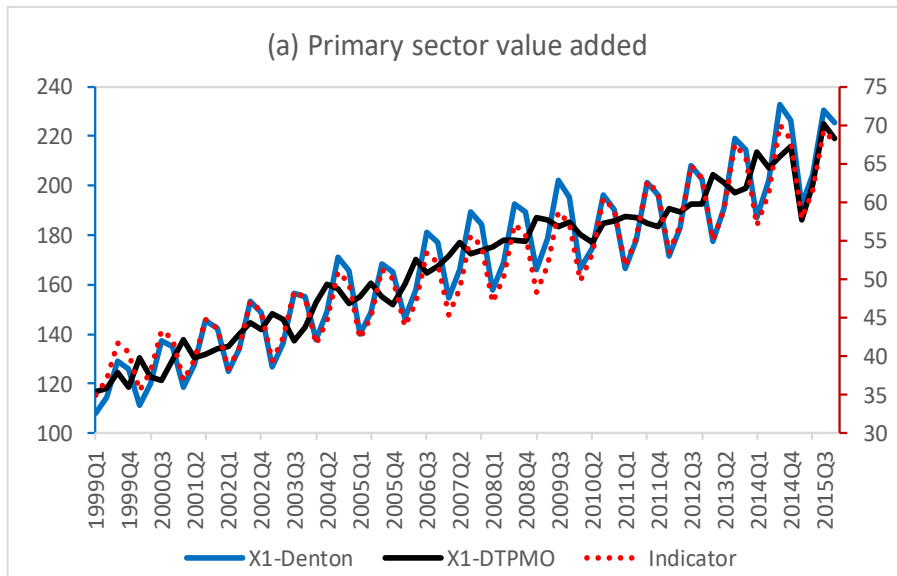
The Pareto boundaries presented in the panels (a2), (b2), (d1) and (d2) indicate that the algorithm converges to a single Pareto optimal point when no\_run=100 for any gen\_max value with pop\_size=100, but the boundaries obtained with gen\_max=25 and gen\_max=100 have the same configuration; the points obtained for the different cases are all located in practically the same restricted space.

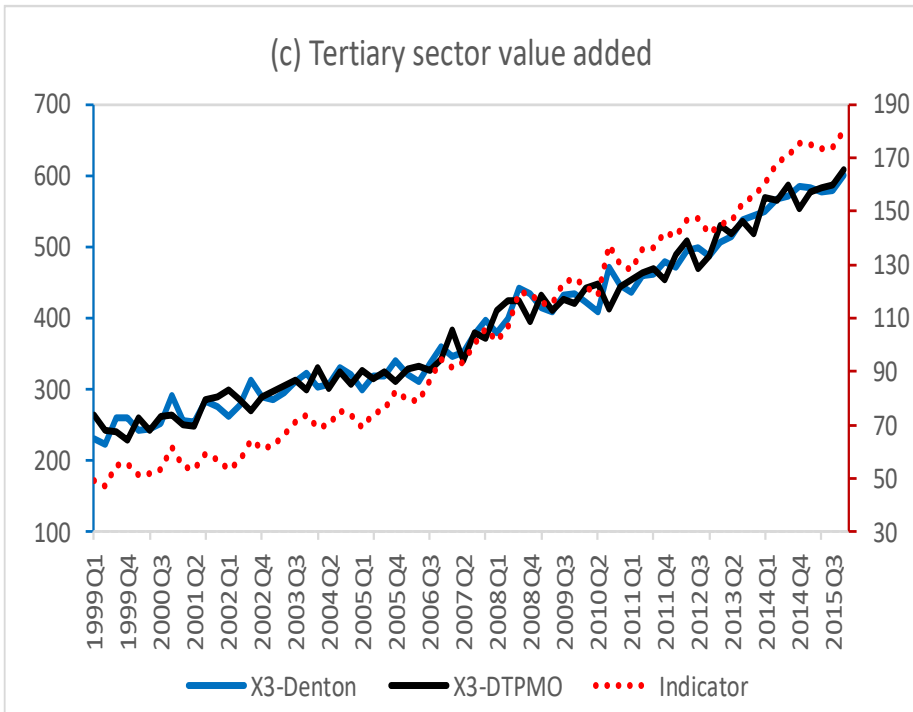
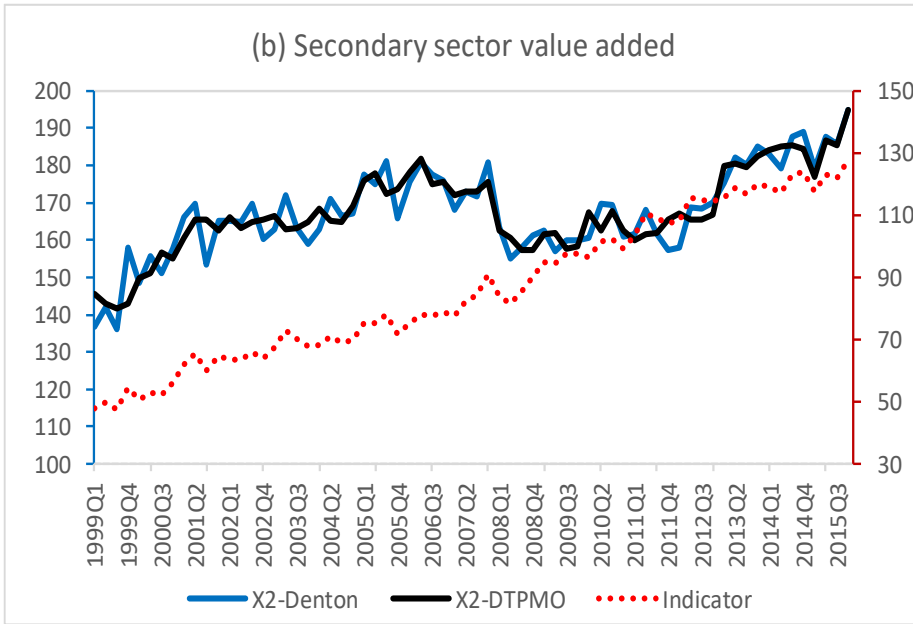
It should be noted that overall, the  $gen\_max$  values set at 25 and 100 give Pareto front with the same characteristics; however, the runtime of the algorithm for displaying results is relatively longer for  $gen\_max=100$  than for  $gen\_max=25$ . Thus, based on the situations presented above, the simulations which of results are presented below are performed with  $(pop\_size, no\_rum, gen\_max) = (100, 10, 25)$ .

### ***Comparison of the evolution of interpolated series***

For the comparison of the model results with those obtained using Denton's proportional method, MOPTD simulations are performed with the parameter  $(pop\_size, no\_runs, gen\_max) = (100,10,25)$ . Figure 2 below presents the results of both methods. The series of quarterly national accounts represented in panels (a), (b), (c) and (d) are those obtained after simulations are made over the period 1999-2014 and extrapolated for the quarters of 2015. The quarterly indicators are represented by red dotted lines. The series of quarterly national accounts obtained with the model are represented in black and those resulting from the application of the Denton's proportional method are the blue curves.

As shown in the results presented in Figure 5, the quarterly national accounts series obtained by both methods and the respective quarterly indicators maintain the trends and relationships obtained with the annual data. However, the analysis of the curves presented in the four panels shows that the MOPTD model produces relatively much smoother (less fluctuating) series compared to the series obtained using Denton's proportional method.





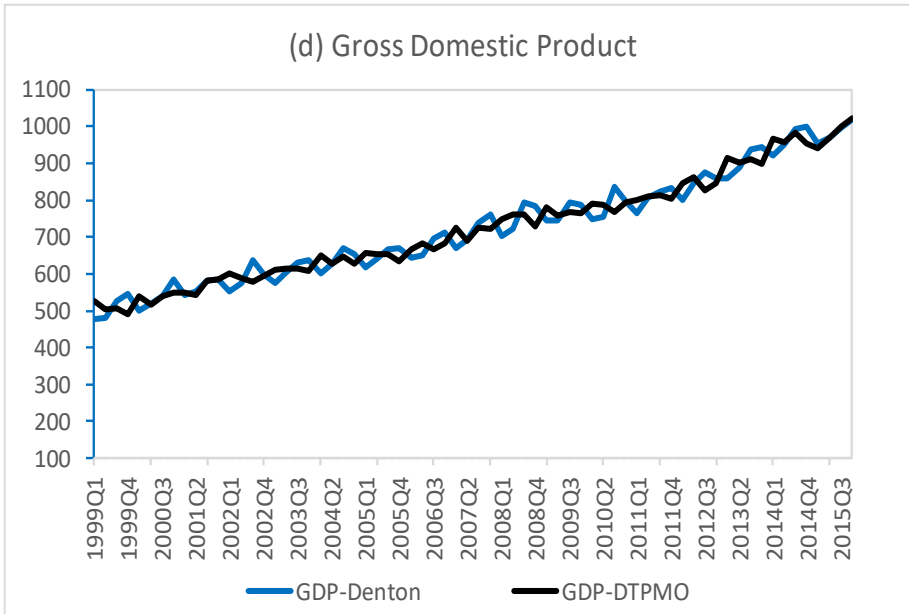


Figure 2 : Comparative evolution of quarterly accounts and indicators

Source: INSAE-data base, Author's calculations

The performance test of the MOPTD model compared to Denton's proportional method is performed in two steps.

The first step in performance analysis is based on a comparison of the level of GDP and growth rate obtained by extrapolation for the four quarters of 2015, after interpolation over the period 1999-2014. Table 1 summarizes the interpolation results compared to the true observed values and Table 2 summarizes the extrapolation results for 2015 in terms of values and growth rates. As the results in Table 1 show, both methods of quarterly disaggregation of GDP replicate exactly the actual observed account values for the retrospective periods (1999-2014).

With regard to the forecast for 2015, as shown in Table 2, for each sector, the annual values obtained by aggregating the quarterly national accounts are different from the real values observed for 2015. The order of magnitude of the difference in values is random, according to an analysis covering the sectors, on the one hand, and the methods, on the other hand. However, by the rule of additivity of accounts and global equilibrium, both methods give the GDP annual values relatively close to the value observed for 2015.

Year	Observed accounts			annual	Annualized accounts-DENTON			quarterly	Annualized accounts-DTPMO			quarterly
	X1	X2	X3	GDP	X1	X2	X3	GDP	X1	X2	X3	GDP

19	477	572	975.	202	477	572	975.	202	477	572	975.	202
99	.64	.96	02	5.63	.64	.96	02	5.63	.64	.96	02	5.63
20	503	612	102	214	503	612	102	214	503	612	102	214
00	.60	.75	7.94	4.28	.60	.75	7.94	4.28	.60	.75	7.94	4.28
20	534	654	107	225	534	654	107	225	534	654	107	225
01	.31	.22	0.11	8.64	.31	.22	0.11	8.64	.31	.22	0.11	8.64
20	561	659	114	236	561	659	114	236	561	659	114	236
02	.24	.70	2.57	3.51	.24	.70	2.57	3.51	.24	.70	2.57	3.51
20	574	657	121	244	574	657	121	244	574	657	121	244
03	.42	.29	3.19	4.90	.42	.29	3.19	4.90	.42	.29	3.19	4.90
20	623	667	126	255	623	667	126	255	623	667	126	255
04	.90	.25	2.06	3.20	.90	.25	2.06	3.20	.90	.25	2.06	3.20
20	622	699	127	259	622	699	127	259	622	699	127	259
05	.49	.62	4.83	6.94	.49	.62	4.83	6.94	.49	.63	4.83	6.94
20	662	710	132	269	662	710	132	269	662	710	132	269
06	.47	.03	6.86	9.36	.47	.03	6.86	9.36	.47	.03	6.86	9.36
20	694	693	147	286	694	693	147	286	694	693	147	286
07	.50	.48	2.97	0.95	.50	.48	2.97	0.95	.50	.48	2.97	0.95
20	708	637	165	300	708	637	165	300	708	637	165	300
08	.50	.89	4.65	1.04	.50	.89	4.65	1.04	.50	.89	4.65	1.04
20	741	639	168	307	741	639	168	307	741	639	168	307
09	.95	.36	9.33	0.64	.95	.36	9.33	0.64	.95	.36	9.33	0.64
20	727	660	174	313	727	660	174	313	727	660	174	313
10	.59	.37	7.60	5.56	.59	.37	7.60	5.56	.59	.37	7.60	5.56
20	742	648	183	322	742	648	183	322	742	648	183	322
11	.19	.71	7.60	8.49	.19	.71	7.60	8.49	.19	.71	7.60	8.49
20	765	664	195	338	765	664	195	338	765	664	195	338
12	.07	.75	4.00	3.82	.07	.75	4.00	3.82	.07	.75	4.00	3.82
20	801	722	210	362	801	722	210	362	801	722	210	362
13	.76	.61	2.79	7.16	.76	.61	2.79	7.16	.76	.61	2.79	7.17
20	848	738	227	386	848	738	227	386	848	738	227	386
14	.41	.97	4.56	1.93	.41	.97	4.56	1.93	.41	.97	4.56	1.93
20	783	851	230	393	852	747	234	394	830	743	235	393
15*	.89	.19	3.51	8.59	.63	.52	0.89	1.05	.99	.72	5.94	0.66

(\*) the account values for this year in the case of Denton and MOPTD are those extrapolated

Table 1 : Comparison of the values of the annual accounts actually observed and aggregated after quarterly reporting

The growth rates resulting from the forecast are also close to the GDP growth rate observed in 2015. The actual observed rate is 1.98% while for the Denton proportional method, the growth rate obtained is 2.05% showing 0.06 point of overrun in percentage.

Annual national accounts	Annual national accounts growth rate
--------------------------	--------------------------------------

	X1	X2	X3	GDP	X1	X2	X3	GDP
Real account value	783.89	851.19	2303.51	3938.59	-7.60%	15.19%	1.27%	1.98%
One year-ahead prediction								
Denton	852.63	747.52	2340.89	3941.05	0.50%	1.16%	2.92%	2.05%
DTMPO								
Simulation 1	830.99	743.72	2355.94	3930.66	-2.05%	0.64%	3.58%	1.78%
Simulation 2	827.30	745.18	2370.19	3942.66	-2.49%	0.84%	4.20%	2.09%
Simulation 3	828.01	744.34	2368.94	3941.29	-2.40%	0.73%	4.15%	2.05%
MEAN	828.77	744.41	2365.02	3938.20	-2.32%	0.74%	3.98%	1.97%

Note : the MOPTD results are those of simulation 1 with (Pop\_size, No\_runs, gen\_max)=(100,10,25)

Table 2 : Summary of extrapolated GDP for 2015

With the MOPTD method, the direction of variation of the GDP growth rate differential obtained by extrapolation, compared to that really observed, is mixed. Indeed, in order to test the robustness of the MOPTD model, the simulation was carried out successively three times with the same parameters. But the random nature conferred on the NSGA-II algorithm induced different values of the GDP extrapolated for 2015. The GDP growth rate extrapolated for 2015 from these three simulations is set at 1.78%, 2.09% and 2.05% respectively, giving an average of 1.97% which is close to the real level observed. This reflects that the MOPTD model is relatively robust.

The second step of performance analysis of the MOPTD model is based on the analysis of statistics on the quality of quarterly series adjustments by interpolation and extrapolation. To do this, the mean coefficient of variation which referred to the mean relative standard deviation (MSD), the mean absolute error (MAE) and the mean absolute revision (MAR) were calculated for the comparison of the performance of the MOPTD model with that of Denton's proportional method.

The standard deviations (SD) of the interpolated quarterly accounts over the period 1999-2014 are calculated in order to compare the volatility of the quarterly value added series X1, X2, X3 and GDP. The MSD resulting from the quarterly GDP adjustment operation is given by the expression (15) :

$$MSD = 0.5 \times \left( \sum_{k=1}^3 SD_{X_k} \bar{w}_k \right) + 0.5 \times SD_{GDP} \quad (15)$$

with  $\bar{w}_k$  the average relative share (over the period 1999-2015) of sector  $k$  value added out of GDP

$$SD_Y = \frac{\sum_{t=1}^{4(T-1)} (Y_t - \bar{Y})^2}{\bar{Y}} ; \bar{Y} \text{ being the average of the series } Y$$

With regard to forecast quality, the analysis of predictive power for each of the two quarterly methods is based on the formula proposed by Marco [18] for calculating the average absolute error. For each series extrapolated to a subsequent year, the estimated prediction error (E) is estimated by calculating the relative difference (in percentage) between the extrapolated value and the true value [18]. In practice, the estimated prediction error of the Y series is given by:

$$\hat{E}_{Y,T} = \frac{\hat{Y}_T - Y_T}{Y_T} \times 100$$

$\hat{Y}_T$  being the estimated value of the series Y (representing either X1, X2, X3 or GDP) by extrapolation for year T (last accounting year). Thus, since the relative difference between the extrapolated and observed of GDP values is not obtained by additivity of errors from the sectors in the GDP calculation, the formula (15) proposed by Marco [18] has been modified and the MAE in predicting quarterly GDP is given by the relationship :

$$MAE = 0.5 \times \left( \frac{1}{3} \sum_{k=1}^3 |\hat{E}_{X_k,T}| \right) + 0.5 \times |\hat{E}_{GDP,T}| \tag{16}$$

In addition, as presented above, the extrapolated values of the account series are different from the true observed values. The annual and quarterly national accounts are therefore subject to revision, due to extrapolation errors, seasonal adjustment or revision of quarterly indicators [16], once the temporary or final annual national accounts are available. As mentioned by Marco [18], it is desirable that the impact of short-term movements be limited so that the quarterly disaggregation method minimizes the size of revisions. Thus, the revision effect is captured by the mean absolute revision (MAR) of the last three years of quarterly changes [18]. The proposed formula for the calculation of MAR is a modification of the one used by Marco [18] and is presented in the following relationship (17):

$$MAR = 0.5 \times \sum_{k=1}^3 \bar{w}_k \left[ \left( \frac{1}{12} \sum_{t=4(T-2)+1}^{4(T+1)} \left| \frac{\hat{X}_{k,t}^{(T+1)}}{\hat{X}_{k,t-1}^{(T+1)}} - \frac{\hat{X}_{k,t}^{(T)}}{\hat{X}_{k,t-1}^{(T)}} \right| \right) \right] + 0.5 \times \frac{1}{12} \sum_{t=4(T-2)+1}^{4(T+1)} \left| \frac{\widehat{PIB}_t^{(T+1)}}{\widehat{PIB}_{t-1}^{(T+1)}} - \frac{\widehat{PIB}_t^{(T)}}{\widehat{PIB}_{t-1}^{(T)}} \right| \tag{17}$$

where the indexes (T + 1) and (T) by exponent represent the last years of observation used;  $\hat{X}_{k,t}$  refers to the value added of the sector k for the quarter t.



The Table 3 presents statistics on the MSD, MAE and MAR for sector value added and GDP. The resulting average values are in bold. The analysis of the results exposed in Table 3 shows overall that the MOPTD model performs relatively better than Denton's proportional method in the quarterly disaggregation of GDP process. Indeed, when reading Table 3, the MOPTD model minimizes the overall MAE and MSD statistics (absolute mean). However, Denton's proportional method minimizes the size of revisions MAR.

	Prediction error (%)	Standard Deviation (%)	Absolute Revision (%)
Primary sector value added (X1)			
Denton	8.77%	18.17%	1.10%
DTPMO	5.72%	16.33%	3.77%
Secondary sector value added (X2)			
Denton	-12.18%	6.69%	1.79%
DTPMO	-12.54%	6.20%	4.07%
Tertiary sector value added (X3)			
Denton	1.62%	26.58%	0.22%
DTPMO	2.67%	26.42%	4.27%
Gross Domestic Product (GDP)			
Denton	0.06%	18.97%	0.05%
DTPMO	-0.01%	18.58%	3.17%
Mean Absolute			
Denton	2.94%	19.41%	0.43%
DTPMO	2.88%	18.90%	3.64%

Note: the DTPMO results are those of simulation 1 with (Pop\_size, No\_runs, gen\_max)=(100,10,25)

Table 3 : Summary of statistics in relation to the accounting forecasting exercise

### Conclusion

The estimation of quarterly GDP using an indirect approach based on quarterly indicators is often done by quarterly disaggregation of the accounts of the various branches that make up GDP. This approach leads to the resolution of several optimization programs. On retrospective data, the quarterly disaggregation of national accounts, based on branch-by-branch disaggregation for the determination of quarterly GDP, does not take into account the link between branches of national accounts in the production process. This research presented a brand new approach to quarterly disaggregate GDP broken down into several branches or sectors that takes into account the links between branches and generalizes the Denton proportional method.

The proposed approach describes the method of quarterly disaggregation of GDP as a quadratic multiobjective programming. In applying the model to Benin's national accounts for which GDP is broken down into three sectors (primary, secondary and tertiary), the resolution was made using a multiobjective evolutionary algorithm using

dominance in the Pareto sense. Since the theoretical formulation is a generalization of Denton's proportional method, the simulation results of the MOPTD model are compared with those obtained by applying Denton's proportional method on the value added of each sector.

It should be noted that the model proposed in this research paper is better suited for the quarterly disaggregation of GDP. Indeed, the quarterly GDP obtained by the MOPTD model is less volatile than that obtained by Denton's proportional method. In addition, on an absolute average, the forecast errors are small for the model compared to that of Denton proportional. However, it should be noted that the Denton method gives small revision errors for series compared to the MOPTD model. This situation could be explained by the random nature of the model's resolution algorithm.

Finally, this research shows that the extrapolations of GDP using the Denton proportional method and those made using the multiobjective programming approach produce similar results when the quarterly indicators used are strongly correlated to the annual accounts. It should be noted that the multiobjective programming approach is better suited to take into account the links between branches of national accounts in the quarterly disaggregation process and to reduce the volatility of the quarterly GDP obtained. The proposed MOPTD model can be applied to the national accounts of any country using the System of National Accounts (SNA) [10] which describes a uniform methodology for the compilation of annual national accounts.

## Acknowledgments

All simulations are performed with Octave software installed on HP Intel Core i7 (vPro) notebook PCs, 16 Gb RAM, under Windows system.

We would like to mention that any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of nor the Institute of Mathematics and Physics Sciences (IMSP) nor the National Institute of Statistics and Economic Analysis.

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