The New Integral Transform: "NE Transform" and Its Applications

Ervenila Musta (Xhaferraj)

Department of Mathematical Engineering, Faculty of Mathematical Engineering and Physics Engineering, Polytechnic University of Tirana, Albania

Abstract

This work introduces a new integral transform for functions of exponential order called "NE integral transform". We prove some properties of NE - transform. Also, some applications of the NE- transform to find the solution to ordinary linear equation are given. The relationships of the new transform with well-known transforms are characterized by integral identities. We study the properties of this transform. Then we compare it with few exiting integral transforms in the Laplace family such as Laplace, Sumudu, Elzaki , Aboodh and etc. As well, the NE integral transform is applied and used to find the solution of linear ordinary differential equations.

Keywords: NE integral transform, Laplas transform, Natural transform , Aboodh transform , Ordinary diferencial equation .

Introduction

For many decades, the integral transforms play a precious role in solving many differential and integral equations. Using an appropriate integral transform helps to reduce differential and integral operators, from a considered domain into multiplication operators in another domain. Solving the deduced problem in the new domain, and then applying the inverse transform serve to invert the manipulated solution back to the required solution of the problem in its original domain (see, [1–13]). The classical integral transforms used in solving differential equations, integral equations, and in analysis and the theory of functions are the Laplace transform, the Fourier integral transform . Besides, in the mathematical literature, there are many Laplace-type integral transforms such as the Laplace-Carson transform which is used in the railway engineering [14], the z-transform can be applied in signal processing [15], the Sumudu transform is used in engineering and many real-life problems [10, 16, 17], the Hankel's and Weierstrass transform has been applied in heat and 1 diffusion equations [18, 19]. In addition, we have the natural transform [20–22] and Yang transform [13, 23] used in many fields of physical science and engineering.

The New Integral Transform

Definition of the transform

This work introduces a new integral transform as a generalization and unification to Laplace and other existing transforms for functions of exponential order . We begin with following: Now let A be the set of single transformable functions that is :

$$A = \left\{ f(t) | \exists M, k_1, k_2 > 0, |f(t)| \leq M\exp\left(\frac{|t|}{k_i}\right) \text{if } t \in (-1)^i x[0, \infty[\right\}$$

The real function f(t) > 0 and f(t) = 0 for t < 0 is sectionwise continuous,exponential order and defined in the set A. The Natural transform of the function f(t) > 0 and f(t) = 0 for t < 0 is defined by :

$$N\{f(t)\} = R(s, u) = \int_0^{+\infty} e^{-st} f(ut) dt \quad , s > 0, u > 0$$

Where s and u are the transform variables. An other integral transform is the Aboodh transform [19] that is derived from equation :

For the same conditions as above is derived equation :

$$A(s) = N\{f(t)\} = \frac{1}{s} \int_0^{+\infty} e^{-st} f(t) dt$$

Definition 2.1

A function f (t) is said to be of exponential order $\frac{1}{k}$, if there exist positive constants T and M such that, | f (t) |≤ $Me^{\frac{t}{k}}$ ( ), for all t ≥ T . For any function f (t), we assume that a integral equation exist.

The New integral transform is the combination of above integral transform .

For a given function in the set A, the constant M must be positiv number. A new integral transform denoted by the operator E(.) is defined by the integral equation:

$E(s, u) = N\{f(t)\} = \frac{1}{su} \int_0^{+\infty} e^{-\frac{st}{u}} f(t) dt$ (1.1) or

$E(s, u) = N\{f(t)\} = \frac{1}{s} \int_0^{+\infty} e^{-st} f(ut) dt$

. (1.1)

Theorem 1.1. *[Sufficient conditions for existence of a new integral transform]*

If f (t) is piecewise continuous on $[0,\infty)$ and of exponential order $\frac{1}{k}$, then E[f (t)] exists for $|\frac{u}{s}| < k$.

Proof. We need to show that the integral

$$E(s,u) = N\{f(t)\} = \frac{1}{su}\int_{0}^{+\infty} e^{-\frac{st}{u}} f(t)dt$$

converges for $|\frac{u}{s}| < k$. We begin by breaking up this integral into two separate integrals:

$$\frac{1}{su}\left[\int_0^T e^{-\frac{st}{u}} f(t)dt + \int_T^{+\infty} e^{-\frac{st}{u}} f(t)dt\right] \qquad (3)$$

where T is chosen so that definition (1.1) holds. The first integral in (3) exists because f (t) and hence $e^{-\frac{st}{u}}$ are piecewise continuous on the interval [0, T) for any fixed s. To see that the second integral in (3) converges, we use the comparison test for improper integrals. Since f (t) is of exponential order $\frac{1}{k}$, we have for t ≥ T , | f (t) |≤ $Me^{\frac{t}{k}}$ and hence:

$$\left|e^{-\frac{st}{u}} f(t)\right| = e^{-\frac{st}{u}}|f(t)| \leq Me^{-t(\frac{s}{u}-\frac{1}{k})}$$

for all t ≥ T.

Now for $|\frac{u}{s}| < k$,

$$\int_{T}^{+\infty} Me^{-t(\frac{s}{u}-\frac{1}{k})}dt = M\int_{T}^{+\infty} e^{-t(\frac{s}{u}-\frac{1}{k})}dt = \frac{Me^{-T(\frac{s}{u}-\frac{1}{k})}}{\frac{s}{u}-\frac{1}{k}} < \infty$$

Since ,

$$\left|e^{-\frac{st}{u}} f(t)\right| \leq Me^{-t(\frac{s}{u}-\frac{1}{k})}$$

for t ≥ T and the improper integral of the larger function converges for $|\frac{u}{s}| < k$, then by the comparison test, the integral $\int_T^{+\infty} e^{-t(\frac{s}{u}-\frac{1}{k})}dt$

converges for $|\frac{u}{s}| < k$. Finally, because the two integrals in (3) exists, a new integral transform N[f (t)] exists for $|\frac{u}{s}| < k$.

Definition 2.2

For the function E(s,u) ,if exist a function f(t) that is piecewise continuous on $[0,+\infty[$, and N(f(t))=E(s,u) , than f(t) is called the invers integral transform of E(s,u) :

$\mathrm{f(t) = N^{-1}(s) = N^{-1}(E\{s,u\}) = \frac{1}{2\pi i}\int_{c-\infty}^{c+\infty} e^{st}E(us)u^2 sds}$ or

$$\mathrm{f(t) = N^{-1}(s) = N^{-1}(E\{s,u\}) = \frac{1}{2\pi i}\int_{c-\infty}^{c+\infty} e^{\frac{st}{u}}E(s)sds}$$

New integral transform of some functions :

$$\mathrm{N\{1\} = \frac{1}{s^2}}$$

$$\mathrm{N\{t\} = \frac{u}{s^3}}$$

$$\mathrm{N\{e^{at}\} = \frac{1}{s(s-au)}}$$

$\mathrm{N\{\mathit{sin}(at)\} = \frac{au}{s(s^2+a^2u^2)}}$

$$\mathrm{N\{\mathit{cos}(at)\} = \frac{1}{(s^2+a^2u^2)}}$$

Theorem 1.2. [Duality relation] Let f (t) ∈ F with Laplace transform F(s). Then a new integral transform E(s,u) of f (t) is given by:

$$\mathrm{E(s,u) = \frac{1}{us}F(\frac{s}{u})}$$

Proof: Let f (t) ∈ F, then for $-k_1 < v < k_2$,

$$\mathrm{E(s,u) = \frac{1}{s^2}\int_0^{\infty} e^{-t}f(\frac{u}{s}t)}$$

Let $\mathrm{v = \frac{u}{s}t}$, then we have:

$\mathrm{E(s,u) = \frac{1}{s^2}\int_0^{\infty} e^{-\frac{sv}{u}}f(v)\frac{s}{u}dv = \frac{1}{us}\int_0^{\infty} e^{-\frac{sv}{u}}f(v)dv = \frac{1}{us}F(\frac{s}{u})}$.

Theorem 1.3. [Fundamental properties of a new integral transform] Let E(s,u) be a new integral transform of f (t). Then:

$N\{f^{'}(t)\} = \frac{sE(s,u)}{u} - \frac{f(0)}{su}$

$N\{f^{''}(t)\} = \frac{s^2E(s,u)}{u^2} - \frac{f(0)}{u^2} - \frac{f^{'}(0)}{su}$

$N\{f^n(t)\} = \frac{s^nE(s,u)}{u^n} - \frac{s^{n-2}}{u^n}f(0) - \frac{s^{n-3}}{u^{n-1}}f^{'}(0) \ldots - \frac{f^{n-1}(0)}{su}$ (3)

$N\{t^n\} = \frac{u^n}{s^{n+2}}\Gamma(n+1)\ ,\ \Gamma(n+1) = (n+1)!\quad (Gamma\ function)$

Proof.

1) For n = 1 and 2 in eqn (4) gives the New transform of first and second derivatives of f(t) respectively. To proceed the induction process,assuming eqn (3) true for n and prove it for n+1,using eqn (1),

$$N[f^{n+1}(t)] = N[f^{n}(t)]' = E_{n+1}(s,u) = \frac{sE_n(s,u)}{u} - \frac{f_n(o)}{us}$$
$$= \frac{s}{u}\left[\frac{s^n}{u^n}E(s,u) - \sum_{k=0}^{n-1}\frac{s^{n-(k+2)}}{u^{n-k}}f^k(0)\right] - \frac{f(0)}{us}$$
$$= \frac{s^{n+1}}{u^{n+1}}E(s,u) - \sum_{k=0}^{n}\frac{s^{n-(k+1)}}{u^{n-k+1}}f^k(0)$$

which is true for n+1 and when n = 0 and 1 in previous relation,gives eqns (1) and (2) respectively.Hence the result (3) follows.

4)

$$N[t^n] = \frac{1}{s}\int_0^\infty e^{-st}(ut)^n dt = \frac{u^n}{s}\int_0^\infty e^{-st}t^n dt = \frac{u^n}{s}\int_0^\infty e^{-v}\frac{v^n}{s^n}\frac{dv}{s} = \frac{u^n\Gamma(n+1)}{s^{n+1}}$$

Here st is replaced with v so that $t = \frac{v}{s}$ and hence ultimately the limit changed and Gamma integral is given by $\Gamma(n) = \int_0^\infty e^{-v}v^{n-1}dv$.

New Integral Transform of Integrals Equations

Consider the integration of function f(t) in set A,w.r.t ′ t ′ in the interval(0,t) as w(t) and successive integrals as $w^2(t)$ upto $w^n(t)$ which is :

$$w(t) = \int_0^t f(t)dt,\quad w^2(t) = \int_0^t\int_0^t f(t)(dt)^2 \quad ,\dots,\quad w^n(t) = \int_0^t \dots \int_0^t f(t)(dt)^n \quad (1.1)$$

Theorem 1.3. New transform of integrals.If $w^n(t)$ is given by (1.1) the New transform of $w^n(t)$ is :

$$N[w^n(t)] = \frac{u^n}{s^n}E(s,u)$$

Proof. From the New integral transform definition eqn (1.1)

$$N[w^n(t)] = \frac{1}{su}\int_0^{+\infty} e^{-\frac{st}{u}}w^n(t)dt = \frac{1}{su}\int_0^{+\infty} e^{-\frac{st}{u}}\left[\int_0^t \dots \int_0^t f(t)(dt)^n\right]dt$$

Applying the integration by parts

$$u = \int_0^t \dots \int_0^t f(t)(dt)^n \ , \quad u^n = f(t)dt \ ,$$

$$dv = e^{-\frac{st}{u}}dt\,, \quad v = -\frac{u}{s}e^{-\frac{st}{u}}\,, v_n = (-1)^n(\frac{u}{s})^n e^{\frac{-st}{u}}$$

$$= \left[(-1)^n \left(\frac{u}{s}\right)^n e^{\frac{-st}{u}} w^n(t)\right]_0^\infty + \frac{1}{su}\int_0^\infty \left(\frac{u}{s}\right)^n e^{\frac{-st}{u}} f(t)dt$$

The first part of the previous equation vanishes and the succeeding integral gives

$$\frac{1}{su}\int_0^\infty \left(\frac{u}{s}\right)^n e^{\frac{-st}{u}} f(t)dt = \left(\frac{u}{s}\right)^n E(s,u) = N[w^n(t)]$$

which ends the proof.

Multiple Shift And Convolucion Theorem

When the function f(t) in set A is multiplied with some shift t then,

$$tf(t) = \sum_{n=0}^{\infty} a_n t^{n+1}$$

The New integral transform of tf(t) gives :

$$N[tf(t)] = \sum_{n=0}^{\infty} \frac{(n+1)!\, a_n u^{n+1}}{s^{n+3}}$$

$$= \frac{u}{s}\sum_{n=0}^{\infty} \frac{(n+1)!\, a_n u^n}{s^{n+2}} = \frac{u}{s}\sum_{n=0}^{\infty} \frac{d}{du}\frac{(n+1)!\, a_n u^{n+1}}{s^{n+2}}$$

$$= \frac{u}{s}\frac{d}{du} u \sum_{n=0}^{\infty} \frac{(n+1)!\, a_n u^n}{s^{n+2}} = \frac{u}{s}\frac{d}{du} uE(s,u).$$

The generalization of previous result is :

Theorem 1.4. The function f(t) in set A is multiplied with shift function t^n then,

$$N[t^n f(t)] = \frac{u^n}{s^n}\frac{d^n}{du^n} u^n E(s,u)$$

Proof : The New transform of Maclaurin series function $f(t) = \sum_{n=0}^{\infty} a_n t^n \in A$ is defined by the infinite series,

$$N[f(t)] = \sum_{n=0}^{\infty} \frac{n!\, a_n u^n}{s^{n+2}}$$

So that we have

$$N[t^n f(t)] = \sum_0^\infty \frac{(2n)!\,a_n u^{2n}}{s^{2n+2}} = \frac{u^n}{s^n}\sum_0^\infty \frac{(2n)!\,a_n u^n}{s^{n+2}} = \frac{u^n}{s^n}\sum_0^\infty \frac{d^n}{du^n}\frac{n!\,a_n u^{2n}}{s^{n+2}}$$
$$= \frac{u^n}{s^n}\frac{d^n}{du^n}u^n\sum_0^\infty \frac{n!\,a_n u^n}{s^{n+2}} = \frac{u^n}{s^n}\frac{d^n}{du^n}u^n E(s,u)$$

Theorem 1.5. If $f^n(t)$ is nth derivative of function f(t) w.r.t ′ t ′ ,is multiplied with shift function t^n then,

$$N[t^n f^n(t)] = u^n \frac{d^n}{du^n}E(s,u)$$

Proof. Differentiating defining integral equation $\frac{1}{s^2}\int_0^\infty e^{-t} f(\frac{ut}{s})dt$, we have :

$$\frac{d^n}{du^n}E(s,u) = \frac{d^n}{du^n}\int_0^\infty \frac{1}{s^2}e^{-t} f\left(\frac{ut}{s}\right)dt = \int_0^\infty \frac{1}{s^2}e^{-t}\frac{\partial^n}{\partial u^n} f\left(\frac{ut}{s}\right)dt = \int_0^\infty \frac{1}{s^2}e^{-t}\left(\frac{t}{s}\right)^n f^n\left(\frac{ut}{s}\right)dt$$
$$= \frac{1}{u^n}\int_0^\infty \frac{e^{-t}}{s}\left(\frac{ut}{s}\right)^n f^n\left(\frac{ut}{s}\right)dt = \frac{1}{u^n}N[t^n f^n(t)]$$

multiplying both sides by u^n ends the proof.

Theorem 1.6. *Convolution theorem.*

If F(s,u) and G(s,u) are the New transforms of respective functions f(t) and g(t) both defined in set A then,

$N[(f * g)] = usF(s,u)G(s,u)$ (2.2)

where $f * g$ is convolution of two functions defined by

$$[(f * g)] = \int_0^t f(a)g(t-a)dt$$

Proof. Expanding the R.H.S of eqn (2.2)

$usF(s,u)G(s,u) =$
$\frac{us}{s}\int_0^\infty e^{-sx} f(ux)dx \frac{1}{s}\int_0^\infty e^{-sy} g(uy)dy = \frac{u}{s}\int_0^\infty e^{-s(x+y)}\int_0^\infty f(ux)g(uy)dxdy$

substituting t = x+y

$$= \frac{1}{s}\int_0^\infty e^{-st}\int_0^\infty f(ux)g\big(u(t-x)\big)udxdy$$

setting a = ux and da = udx with ux in [0,ut] and x is in [0,t] thus

$$= \frac{1}{s}\int_0^\infty e^{-st}\int_0^t f(ux)g\big(u(t-x)\big)d(ux)dt = \frac{1}{s}\int_0^\infty e^{-st}\int_0^{ut} f(a)g(ut-a)dadt = N[f*g]$$

The New Decomposition Method

In this section, we illustrate the applicability of the New Decomposition Method to some nonlinear ordinary differential equations.

Consider the general nonlinear ordinary differential equation of the form:

$Lv + R(v) + F(v) = g(t)$, (4.1)

subject to the initial condition $v(0) = h(t)$, (4.2)

where L is an operator of the highest derivative, R is the remainder of the differential operator, $g(t)$ is the nonhomogeneous term and $F(v)$ is the nonlinear term. Suppose L is a differential operator of the first order, then by taking the New Transform of Eq. (4.1), we have:

$\frac{sE(s,u)}{u} - \frac{V(0)}{us} + N[R(v)] + N[F(v)] = N[g(t)]$ (4.3)

By substituting Eq. (4.2) into Eq. (4.3), we obtain:

$E(s,u) = \frac{h(t)}{s^2} + \frac{u}{s}N[g(t)] - \frac{u}{s}N[R(v)] - \frac{u}{s}N[F(v)]$ (4.4)

Taking the inverse of the New Transform of Eq. (4.4), we have:

$$v(t) = G(t) - N^{-1}\left[\frac{u}{s}N[R(v) + F(v)]\right] \qquad (4.5)$$

where $G(t)$ is the source term.

We now assume an infinite series solution of the unknown function $v(t)$ of the form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \qquad (4.6)$$

Then by using Eq. (4.6), we can re-write Eq. (4.5) in the form:

$$\sum_{n=0}^{\infty} v_n(t) = G(t) - N^{-1}\left[\frac{u}{s}N\left[R\sum_{n=0}^{\infty} v_n(t) + \sum_{n=0}^{\infty} A_n(t)\right]\right] \qquad (4.7)$$

where $A_n(t)$ is an Adomian polynomial which represent the nonlinear term. Comparing both sides of Eq. (4.7), we can easily build the recursive relation as follows:

$$v_0(t) = G(t)$$

$$v_1(t) = -N^{-1}\left[\frac{u}{s}N\left[R\big(v_0(t)\big) + A_0(v)\right]\right]$$

$$v_2(t) = -N^{-1}\left[\frac{u}{s}N\left[R\big(v_1(t)\big) + A_1(v)\right]\right]$$

$$v_3(t) = -N^{-1}\left[\frac{u}{s}N\left[R\big(v_2(t)\big) + A_2(v)\right]\right]$$

Eventually, we have the general recursive relation as follows:

$$v_{n+1}(t) = -N^{-1}\left[\frac{u}{s}N\left[R\big(v_n(t)\big) + A_n(v)\right]\right] \qquad n \geq 0 \quad (4.8)$$

Hence, the exact or approximate solution is given by:

$v(t) = \sum_{n=0}^{\infty} v_n(t)$ (4.9)

Applications to Find the Solution of Linear Ordinary Equations

Example 1 :

Consider the Riccati differential equation of the form :

$\frac{dv}{dt} = 1 - t^2 + v^2(t)$ (5.1)

subject to the initial condition $v(0) = 0$. (5.2)

Taking the New Transform to both sides of Eq. (5.1), we obtain:

$\frac{sE(s,u)}{u} - \frac{v(0)}{us} = \frac{1}{s^2} - \frac{2u^2}{s^4} + N[v^2(t)]$ (5.3)

By substituting Eq. (5.2) into Eq. (5.3), we obtain:

$$v(s,u) = \frac{u}{s^3} - \frac{2u^3}{s^5} + \frac{u}{s}N[v^2(t)] \qquad (5.4)$$

Taking the inverse New Transform of Eq. (5.4), we have:

$$v(t) = t - \frac{t^3}{3} + \frac{u}{s}N[v^2(t)] \qquad (5.5)$$

We now assume an infinite series solution of the unknown function v(t) of the form:

$$v(t) = \sum_{n=0}^{\infty} v_n(t) \qquad (5.6)$$

Then by using Eq. (5.6), we can re-write Eq. (5.5) in the form:

$\sum_{n=0}^{\infty} v_n(t) = t - \frac{t^3}{3} + N^{-1}\left[\frac{u}{s}N[\sum_{n=0}^{\infty} A_n(t)]\right]$ (5.7) where A_n is the Adomian polynomial which represent the nonlinear term $v^2(t)$. By comparing both sides of Eq. (5.7), we can easily build the general recursive relation as follows:

$$v_0(t) = t - \frac{t^3}{3}$$

$$v_1(t) = N^{-1}[\frac{u}{s}N[A_0(t)]]$$

$$v_2 = N^{-1}[\frac{u}{s}N[A_1(t)]]$$

Then the general recursive relation is given by:

$$v_{n+1}(t) = N^{-1}[\frac{u}{s}N[A_n(t)]]. \quad (5.8)$$

By using Eq. (5.8), we can easily compute the remaining components of the unknown function v(t) as follows:

$$v_1(t) = N^{-1}\left[\frac{u}{s}N[A_0(t)]\right]$$

$$= N^{-1}[\frac{u}{s}N[v_0^2(t)]]$$

$$= N^{-1}[\frac{u}{s}N[((t - \frac{t^3}{3})^2)]]$$

$$= N^{-1}[\frac{u}{s}N[t^2]] - \frac{2}{3}N^{-1}\left[\frac{u}{s}N[t^4]\right] + \frac{1}{9}N^{-1}[\frac{u}{s}N[t^6]]$$

$$= N^{-1}\left[\frac{2u^3}{s^4}\right] - \frac{2}{3}N^{-1}\left[\frac{4!\,u^5}{s^6}\right] + \frac{1}{9}N^{-1}[\frac{6!\,u^7}{s^8}]$$

$$= \frac{t^3}{3} - \frac{2t^5}{15} + \frac{t^7}{63}$$

Then by canceling the noise term from $v_0(t)$, the remaining non-canceled term of $v_0(t)$ provide us with the exact solution. This can easily be verified by substitution.

Therefore, the exact solution of the given problem is given by:

$v(t) = t.$ (5.9)

The exact solution is in closed agreement with the result obtained by (ADM).

Example 2 :

The following examples illustrate the use of a new integral transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation:

$y'(t) + by(t) = h(t), \quad t > 0, \; y(0) = a \quad (1)$

where a and b are constants and h(t) is an external input function so that its a new integral transform exists.

Using a new integral transform of equation (1) we have:

$$\frac{sE(s,u)}{u} - \frac{y(0)}{us} + bE(s,u) = H(s,u)$$

where that E(s,u) and H(s,u) are a new integral transforms of y(t) and h(t).

By applying the initial condition we have:

$$E(s,u)\left[\frac{s}{u} - b\right] = H(s,u) + \frac{a}{us}$$

$$= \frac{u(usH(s,u) + a)}{us(s - bu)}$$

$$\Rightarrow E(s,u) = a\left[\frac{1}{s(s - bu)}\right] + us\left[\frac{H(s,u)}{s(s - bu)}\right]$$

By the inverse of a new integral transform and convolution theorem we find that:

$y(t) = ae^{-bt} + \int_0^{\tau} e^{-bt} h(t-\tau)d\tau \qquad (2)$

The first term of this solution in (2) is independent of time t and is usually called the steady-state solution, and the second term depends on time t and is called the transient solution. In the limit as $t \to \infty$ the transient solution decays to zero if $b > 0$ the steady-state solution is attained, on the other hand, when $b < 0$, the transient solution grows exponentially as $t \to \infty$ and the solution becomes unstable. Equation (2) describes the Law of natural growth or decay Process with an external forcing function h(t) according as $b > 0$ or $b < 0$. In particular, if $h(t) = 0$ and $b > 0$ the resulting equation (2) occurs very frequently in chemical kinetics. Such an equation describes the rate of chemical reactions.

Conclusion

In the present paper, authors successfully introduced a new integral transform “NE transform” and analyzed the novel integral transform. . Authors also presented the fundamental properties (linearity; scaling; translation; convolution) of the proposed transform with its inverse transform. The definition and applications of the novel transform to solve ordinary differential equations have been demonstrated . In future, “NE integral transform” can be considered to solve various complex problems of science, medicine and engineering by developing their mathematical models, also we will introduce the complex transform and a double” NE integral transform “.

References

[1] M. Akel, H. M. Elshehabey, R. Ahmed, Generalized laplace-type transform method for solving multilayer diffusion problems, Journal of Function Spaces, vol. 2022, Article ID 2304219, 20 pages, 2022.

[2] R. Aruldoss and K. Balaji, Numerical inversion of Laplace transform via Wavelet operational matrix and its applications to fractional differential equations, Int. J. Appl. Comput. Math., (2022), 8–16.

[3] M. Abdalla and M. Akel, Contribution of using Hadamard fractional integral operator via Mellin integral transform for solving certain fractional kinetic matrix equations, Fractal Fract., 6 (2022), 1–14.

[4] M. Abdalla, S. Boulaaras and M. Akel, On Fourier-Bessel matrix transforms and applications, Mathematical Methods in the Applied Sciences., 44, (2021), 11293– 11306.

[5] L. Boyadjiev and Y. Luchko, Mellin integral transform approach to analyze the multidimensional diffusion-wave equations, Chaos. Solitons. Fractals., 102, (2017) 127–134.

[6] R. M. Cotta, Integral transforms in computational heat and fluid flow, CRC Press, 2020

[7] M. Consuelo Casaban, R. Company, V. Egorova, and L. Jodar, Integral transform solution of random coupled parabolic partial differential models, Mathematical Methods in the Applied Sciences., 43, (2020), 8223 - 8236.

[8] B. Davis, Integral Transforms and Their Applications, 3rd ed.; Springer: New York, NY, USA, 2002.

[9] L. Debnath and D. Bhatta, Integral Transforms and Their Applications, Third Edition, Chapman and Hall (CRC Press), Taylor and Francis Group, London and New York, 2016.

[10] Q. D. Katatbeh and F. B. M. Belgacem, Applications of the sumudu transform to fractional differential equations, Nonlinear Studies., 18, (2011) 99 - 112.

[11] M. Hidan, M. Akel, S. Boulaaras and M. Abdalla, On behavior Laplace integral operators with generalized Bessel matrix polynomials and related functions, vol. 2021, Article ID 9967855, 10 pages, 2021.

[12] M. R. Rodrigo and A. L. Worthy, Solution of multilayer diffusion problems via the laplace transform, Journal of Mathematical Analysis and Applications., 444, (2016), 475–502. 23

[13] X.-J. Yang, F. Gao, Y. Ju and H.-W. Zhou, Fundamental solutions of the general

fractional-order diffusion equations, Mathematical Methods in the Applied Sciences 41, (2018), 9312 – 9320

[14] M. L´evesque, M. D. Gilchrist, N. Bouleau, K. Derrien and D. Baptiste, Numerical inversion of the Laplace-Carson transform applied to homogenization of randomly reinforced linear viscoelastic media, Computational mechanics., 40, (2007), 771–789.

[15] Y.-L. Cui, B. Chen, R. Xiong and Y.-F. Mao, Application of the z-transform technique to modeling the linear lumped networks in the hie-fdtd method, Journal of Electromagnetic Waves and Applications., 27, (2013), 529–538.

[16] H. Bulut, H. M. Baskonus and F. B. M. Belgacem, The analytical solution of some fractional ordinary differential equations by the sumudu transform method, in: Abstract and Applied Analysis, vol. 2013, Article ID 203875, 6 pages, 2013.

[17] F. Belgacem and A. Karaballi, Sumudu Transform Fundamental Properties Investigations and Applications, Journal of Applied Mathematics and Stochastic Analysis., vol. 2006, Article ID: 91083, 23 pages, 2006.

[18] Z. H. Khan, W. A. Khan, Natural transform-properties and applications, NUST J. Eng. Sci., 1 (2008), 127–133.

[19] K. S. Aboodh, The new integral transform “Aboodh transform”, Global Journal of Pure and Applied Mathematics, 9 (2013), 35–43.